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Combinatorial Optimization

Exercise 1 (Fractional Knapsack)

Let $c, w, \in \mathbb{N}^n$ be non-negative integral vectors with

$$\frac{c_1}{w_1} \geq \frac{c_2}{w_2} \geq \cdots \geq \frac{c_n}{w_n}$$

and let

$$k = \min\left\{ j \in \{1, \ldots, n\} : \sum_{i=1}^{j} w_i > W \right\}.$$

Show that an optimum solution for the Fractional Knapsack problem is given by

$$\begin{aligned} x_j &= 1 \quad \text{for } j = 1, \ldots, k-1, \\ x_j &= \frac{W - \sum_{i=1}^{k-1} w_i}{w_k} \quad \text{for } j = k, \text{ and} \\ x_j &= 0 \quad \text{for } j = k+1, \ldots, n. \end{aligned}$$

Exercise 2 (Greedy Knapsack)

Construct an instance for which the approximation-guarantee 1/2 of the Greedy algorithm for Knapsack is achieved.

Exercise 3 (Greedy Set Cover)

Construct an instance for which the approximation-guarantee H_n of the Greedy algorithm for Set Cover is achieved.

Exercise 4 (Cardinality Vertex Cover)

Let $G = (V, E)$ be a graph. A subset $C \subseteq V$ is called a *vertex cover* if each edge $e \in E$ is incident to a vertex $v \in C$. The problem Cardinality Vertex Cover asks to find a vertex cover with as few vertices as possible. This problem is NP-hard.

Show that the following simple algorithm is a 2-approximation: Compute a matching $M \subseteq E$ in G which is maximal with respect to inclusion and include all the vertices v that are incident to an edge $e \in M$ in the cover C.