Albert-Ludwigs-Universität
Institut für Informatik
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Algorithms Theory, Winter Term 07/08
Assignment 1

hand in by Monday, November 5, 2007, 14 p.m.
(boxes in building 051)

Exercise 1: Minimum and maximum (5 points)
Let $S = \{x_1, \ldots, x_n\}, n = 2^i, i \in \mathbb{N}, i \geq 1$, be a set of natural numbers. We wish to determine the minimum and the maximum in S. How many comparisons are necessary in an iterative approach? Describe the corresponding procedure and argue how many comparisons are required. Then, use the divide-and-conquer paradigm for developing an algorithm that identifies both values using at most $\frac{3}{2}n - 2$ comparisons. Describe the algorithm in words or in pseudocode and verify the number of comparisons by induction.

Exercise 2: Closest pair (5 points)
For the set $S = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$ of points with

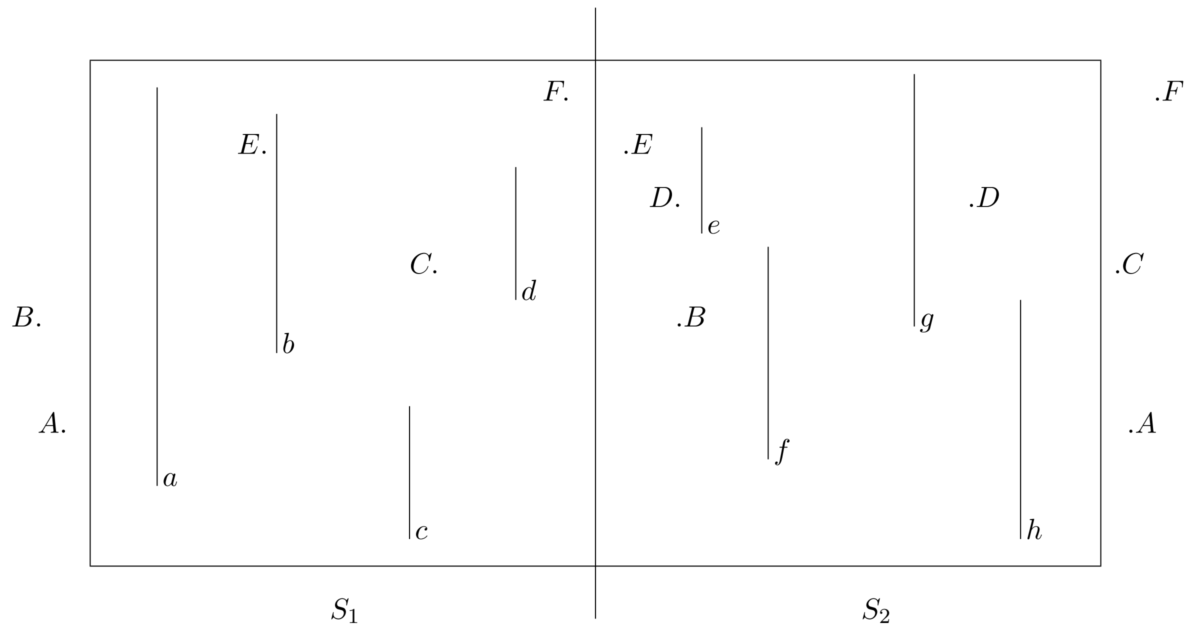
$$p_1 = (3,4), p_2 = (7,9), p_3 = (2,8), p_4 = (5,7),$$

$$p_5 = (10,3), p_6 = (0,5), p_7 = (8,0), p_8 = (7,2)$$

find a *closest pair* using the strategy discussed in the lectures. Specify all recursive calls of `mindist()`, the corresponding results for S_l, S_r, d_l, d_r, the bounds d used in the respective merge steps, as well as the closest pair and the corresponding distance for each call.

Exercise 3: Geometric divide-and-conquer (5 points)
Let $a, \ldots, h$ be some vertical line segments and $A, \ldots, F$ be some horizontal line segments given by the corresponding left and right endpoints (i.e. denoted by $A.$ and $.A$, respectively). Furthermore, let S be the set of all vertical line segments and of all pairs of endpoints of the horizontal line segments. Assume that all pairs of intersecting line segments should be determined using the divide-and-conquer procedure. The divide step yields the partitioning below. There is $S = S_1 \cup S_2$. Specify the output of the two calls `ReportCuts(`S_1`)` and `ReportCuts(`S_2`)`, the sets $L(S_1)$, $L(S_2)$, $R(S_1)$, $R(S_2)$, $V(S_1)$, $V(S_2)$, $L(S)$, and $R(S)$, as well as the intersections identified in the merge step. By means of these sets, determine the output of `ReportCuts(`S`)`.

Notation: $L(S) = \{P | P. \in S \wedge .P \notin S\}$, $R(S) = \{P | P. \notin S \wedge .P \in S\}$ and $V(S) = \{p \in S\}$. The intersection of a horizontal line segment P with a vertical line segment p is denoted as the ordered pair (P, p). Therefore, the set `ReportCuts(`S`)` consists auf ordered pairs (P, p).



Exercise 4: Roots of unity (5 points)

a) Determine all 8th roots of unity and illustrate them in the plane of complex numbers. Write the roots of unity as $a + b\,i$ with $a, b \in \mathbb{R}$.

b) Specify all primitive 8th roots of unity. Show that they are indeed primitive.

Hint: An nth root of unity ω is *primitive* if $\omega^k \neq 1$ for $k = 1, \cdots, n-1$. Thus, a primitive root of unity ω generates the group of nth roots of unity:

$$\{\omega^k : 0 \leq k \leq n-1, k \in \mathbb{N}\} = \{\omega_n^0, \omega_n^1, \cdots, \omega_n^{n-1}\}.$$

Information on the course:

Students are eligible to participate in the final exam if they have

- completed 50 % of the exercises
- presented 1 exercise during the exercise sessions

The final grade will be upgraded by

- 1/3 grade if the student has achieved 50 % of the points attainable in the exercises
- 2/3 grade if the student has achieved 80 % of the points attainable in the exercises

The solutions to the exercises can be submitted in English as well as in German.