Albert-Ludwigs-Universität
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Algorithms Theory, Winter Term 07/08
Assignment 4

hand in by Monday, December 17, 2007, 14 p.m.
(boxes in building 051)

Exercise 1: Potential method (5 points)

a) Suppose we have a potential function Φ such that $\Phi_0 \neq 0$ and $\Phi_i \geq \Phi_0$ for all i. Show that there exists a potential function Φ' such that $\Phi'_0 = 0, \Phi'_i \geq 0$ for all $i \geq 1$, and the total amortized costs using Φ' are the same as the total amortized costs using Φ.

b) Suppose that a binary counter starts at a number with b 1's in its binary representation, rather than at 0. Show that the cost of performing n `increment` operations is $O(n)$ given that $n \geq b$.

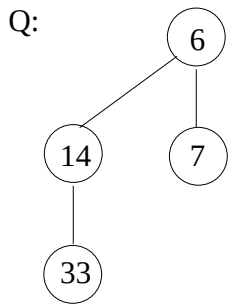
Exercise 2: Octal counter (5 points)
Consider a counter to base 8 that counts upward in the octal number system starting at 0. The cost for incrementing the counter by 1 equals the number of digits that have to be changed. Let $A(n)$ be the cost for n successive increments.

a) Use the accounting method to argue that for all $n \in \mathbb{N}$ there is $A(n) \leq \frac{8}{7}\, n$.

b) Show that $\frac{8}{7}$ is optimal in that for each c satisfying $A(n) \leq c \cdot n$ for all $n \in \mathbb{N}$ it must hold $c \geq \frac{8}{7}$.

Exercise 3: Binomial queues (5 points)
Execute the following operations on the binomial queue below. For each intermediary stage illustrate the current state of the queue. At the end, plot the pointer references used in the child-sibling representation of the resulting queue.

$Q.insert(15)$, $Q.insert(10)$, $Q.insert(22)$, $Q.insert(3)$, $Q.insert(28)$, $Q.insert(20)$
$Q.deletemin()$, $Q.decreasekey(20, 1)$



Exercise 4: Child-sibling representation (5 points)
Suppose that in the child-sibling representation $child[x]$ points to the *leftmost* child of node x and $sibling[x]$ points to the sibling of x *immediately to the right*.

a) Suppose that x is a node in a binomial tree within a binomial queue, and assume that $sibling[x]$ is not null. If x is not a root, how does $degree[sibling[x]]$ compare to $degree[x]$? How about if x is a root?

b) If x is a nonroot node in a binomial tree within a binomial queue, how does $degree[x]$ compare to $degree[parent[x]]$?