Albert-Ludwigs-Universität
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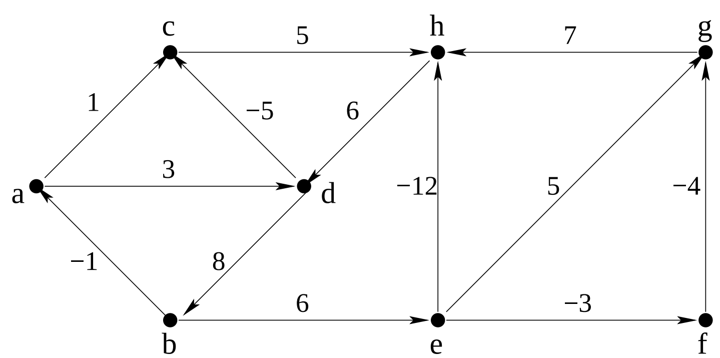
Algorithms Theory, Winter Term 07/08
Assignment 6

hand in by Monday, January 28, 2008, 14 p.m.
(boxes in building 051)

Exercise 1: Greedy strategy for activity selection (5 points)
Suppose that instead of always selecting the activity with the earliest finish time, we select the activity with the latest start time that is compatible with all previously selected activities. Why is this approach a greedy strategy? Adjust algorithm *Greedy-Activity-Selector* appropriately and, just as in the lecture, prove by induction that it yields an optimal solution.

Exercise 2: Shortest paths (5 points)
Consider the directed graph G shown below. Execute the Bellman-Ford algorithm in order to determine the shortest paths from vertex a to all other vertices. For each execution of the *while*-loop specify all intermediary stages of the queue U, as well as the values $DIST[\cdot]$ and $A[\cdot]$ that are changed. Finally, give the resulting arrays $DIST$ and A and plot the subgraph of G consisting of all edges that constitute the shortest paths.

Remark: If, during an execution of the *while*-loop, there are several vertices to be appended at U, then they should be considered in lexicographic order.



Exercise 3: Minimum spanning trees (5 points)
Let T be a minimum spanning tree of a graph $G = (V, E)$ with a weight function $w : E \to \mathbb{R}$, and let V' be a subset of V. Let T' be the subgraph of T induced by V', and let G' be the subgraph of G induced by V'. Show that if T' is connected, then T' is a minimum spanning tree of G'.

Exercise 4: Bin packing (5 points **+ 3 extra points**)

In the lecture, an input sequence for the bin packing problem was given, for which the number of bins used by *First Fit* is $\frac{10}{6}$ times the number of bins required in an optimal packing. Now, consider the following extension of this sequence:

$$\underbrace{\frac{1}{43}+\epsilon,\ldots,\frac{1}{43}+\epsilon}_{42m}\,,\ \underbrace{\frac{1}{7}+\epsilon,\ldots,\frac{1}{7}+\epsilon}_{42m}\,,\ \underbrace{\frac{1}{3}+\epsilon,\ldots,\frac{1}{3}+\epsilon}_{42m}\,,\ \underbrace{\frac{1}{2}+\epsilon,\ldots,\frac{1}{2}+\epsilon}_{42m} \qquad (m \in \mathbb{N})$$

a) Draw an optimal packing as well as the resulting packing of *First Fit*. What is the number of bins used by *First Fit* compared to the optimum number of bins?

b) How does the offline strategy *First Fit Decreasing* proceed regarding the sequence above? What is the resulting packing and how many bins are used?

The following subtask gives 3 extra points:

c) How could you extend the input sequence such that the number of bins used by *First Fit* is even worse compared to the optimum number of bins? Try to find a general design pattern and a general expression for the corresponding numbers of bins used.