
First Assignment

Selected Topics in Efficient Algorithms

To be returned in the lectures on November, 6th 2007.

Exercise 1: A railroad company offers a discount card enabling you to pay only 50% of the original ticket price. The card costs 200 bucks. The problem is that you do not know the number of your future train journeys.

- Give an optimal offline strategy for this problem.
- Give a lower bound for any online algorithm for this problem.
- Give an online algorithm that matches your lower bound.
- What is the optimal offline strategy if the company additionally introduces a card at a cost of 50 bucks which gives you 25% discount.

Exercise 2: Consider the following problem. We want to schedule n jobs on m identical machines. Every job $i \in \{1, \dots, n\}$ can be scheduled on any processor and needs t_i time units to be completed. If a job is scheduled on a processor it has to be processed there without interruption. Our goal is to schedule all jobs to the m processors with respect to the objective function

$$\min_{1 \leq j \leq m} l_j \rightarrow \max$$

where l_j is the sum of all t_i of the jobs i that are scheduled on machine j .

Give an adequate definition of the competitive ratio for *maximization* problems and prove the highest lower bound you can find for the competitive ratio of any online algorithm for this problem.

Exercise 3: Show that the **FIFO** paging algorithm is k -competitive, where k is the size of the fast storage.

Exercise 4: Show that the **LIFO** paging algorithm is **not** k -competitive for any k , where k is the size of the fast storage.