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Fifth Assignment

Selected Topics in Efficient Algorithms

To be returned in the lectures on January 15th, 2008.

Exercise 1: Prove Observation 4.1 from the lectures, which states that for every s-t-flow we have

$$\sum_{e^-=s} f(e) - \sum_{e^+=s} f(e) = \sum_{e^+=t} f(e) - \sum_{e^-=t} f(e).$$

Exercise 2: Let $G = (V, E)$ be a directed graph with capacities $c : E \to \mathbb{N}_0$. We introduce a new capacity function

$$c'(e) := |E| \cdot c(e) + 1 \quad \text{for all } e \in E.$$

Show that if (S, T) is a minimal cut relative to c' then (S, T) is minimal relative to c, too. Furthermore (S, T) has the minimal number of edges among all cuts relative to c.

Exercise 3: A commander is located at one vertex p in an undirected communication network G and his subordinates are located at vertices denoted by the set S. Let u_{ij} be the effort required to eliminate edge ij from the network. The problem is to determine the minimal effort required to block all communications between the commander and his subordinates. How can you solve this problem in polynomial time?

Exercise 4: Christmas dinner is impending and q tables are layed. There are p families to join the big dinner and the ith family has $a(i)$ members. Furthermore table j has $b(j)$ seats. Since families are always at odds with themselves on christmas they want to sit at tables so that no two members of the same family are at the same table. Show how to formulate finding a seating arrangement that meets this objective as a maximum flow problem.

Exercise 5: Santa Claus wants to distribute his presents all over the country. For this purpose he provides a logistic network with storage vertices and transportation edges forming a graph. Of course there are capacities given on the transportation links as usual, but additionally we have a maximum number of packets that can be handled by a storage center. This means, we have capacities on the vertices, too. Santa's objective is to maximize the throughput of presents respecting the capacities of transportation links and logistic centers. Transform this problem to the standard maximization flow problem. From the perspecitve of worst-case complexity, is the maximum flow problem with upper bounds on vertices more difficult to solve that the standard maximum flow problem?

Please visit the following link frequently for ongoing information:
`http://www.informatik.uni-freiburg.de/~ipr/ipr_teaching/ws07_08/selected_topics.html`