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Sixth Assignment

Selected Topics in Efficient Algorithms

To be returned in the lectures on January 29th, 2007.

Exercise 1: Are the following assertions correct?

1. Every LP with at least one feasible solution has also an optimal solution.
2. There is an LP with more than one but only finitely many optimal solutions.

Give explanations for your answers in both cases.

Exercise 2: Consider the following LP:

$$\begin{array}{rl} \max & x_1 + x_2 \\ \text{subject to} & sx_1 + tx_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{array}$$

Find necessary and sufficient conditions on s and t such that this LP

1. has at least one optimal solution,
2. has exactly one optimal solution,
3. has no solution,
4. is unbounded.

Exercise 3: Let $G = (V, E)$ be a directed graph with edge capacities $c : E \to \mathbb{R}^+$ and $s, t \in V$ two specified nodes. Moreover denote by

$$S = \{P \subseteq E : P \text{ is } (s-t)\text{-path}\}$$

the set of all (s, t)-paths in G.

Consider the following LP:

$$\begin{array}{rl} \max & \displaystyle\sum_{P \in S} x_P \\ \text{subject to} & \displaystyle\sum_{\substack{P \in S \\ e \in P}} x_P \leq c_e \quad \forall e \in E \\ & x_P \geq 0 \quad \forall P \in S \end{array}$$

1. Give the dual program to this LP.
2. Give the graph theoretical interpretation of the LP and its dual.

[Please turn over!]

Exercise 4: Let (P) and (D) denote the primal and dual of an LP in standard form. Let x and y be feasible solutions for (P) and (D), respectively. Then x and y are optimal solutions of (P) and (D) if and only if

1. $y_i \geq 0 \Rightarrow \sum_{j=1}^{n} a_{ij} x_j = b_i \quad (i = 1, \ldots, m)$, and
2. $x_j \geq 0 \Rightarrow \sum_{i=1}^{m} a_{ij} y_i = c_i \quad (j = 1, \ldots, m)$.