Albert-Ludwigs-Universität
Institut für Informatik
Prof. Dr. Susanne Albers
Dr. Alexander Souza
Fabian Schiller

Freiburg i. Br., January 29th, 2008
Winter Term 07/08

Seventh Assignment

Selected Topics in Efficient Algorithms

To be returned in the exercises on February 12th, 2007.

Exercise 1: Let the following LP (I) in canonical form be given:

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

The goal of this exercise is to find a feasible solution for this LP.

1) Construct another LP (II) in canonical form with the following properties:
   a) The LP (I) is feasible, if and only if LP (II) has optimum value equal to zero.
   b) From any optimum solution of LP (II) a feasible solution of LP (I) can be read off.
   c) A feasible solution for LP (II) is easily seen.
2) In terms of computational complexity, what is more difficult: optimizing an LP or finding a feasible solution? Justify your answer.

Exercise 2: Prove a lower bound of 2 for the approximation factor of algorithm MST from the lectures. For this purpose consider only graphs for which the triangle inequality holds.

Exercise 3: Let G be a complete undirected graph in which all edge lengths are either 1 or 2. Give a 4/3-approximation algorithm for TSP in this special class of graphs.
Hint: Start by finding a minimum 2-matching in G. A 2-matching is a subset S of edges so that every vertex has exactly 2 edges of S incident to it.

Exercise 4: Consider the following algorithm for Minimum Makespan Scheduling: The algorithm arranges the jobs in according to decreasing processing times and schedules them in this order in greedy manner. Give a lower bound strictly larger than one for the approximation ratio of this algorithm.

Exercise 5: Let A be a Minimum Makespan Scheduling algorithm with approximation factor α in the setting where every job is available at time 0. Assume now, that every job j has a release time r_j and may only be scheduled after this time. Show that the following is a 2α-approximation algorithm.

The new algorithm A' schedules the jobs in phases. Let S_{i+1} be the set of jobs released in $(F_{i-1}, F_i]$, and let F_i be the point in time, where schedule S_i completes. At time F_i the scheduler uses algorithm A to schedule the jobs in S_{i+1}.