Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Prof. Dr. Susanne Albers
Dr. Alexander Souza

Winter Term 2008/09
Sheet 02

Average-Case Analysis

Exercise 1 (Let's Make A Deal – Should I Say Or Should I Go?)

In the gameshow "Let's Make A Deal" there are three closed doors, left, middle, and right. Only behind one of them there is a price, the others are blank. The participant – you – initially specifies a door. Then the quizmaster opens one of the blank doors and asks you if you want to stick to your door or switch to the remaining closed one.
Which strategy is better? Give the probabilities for winning the price in both strategies (stay and switch) assuming you have chosen your initial door uniformly at random.

Exercise 2 (Fractional Knapsack)

Let $c, w, \in \mathbb{N}^n$ be non-negative integral vectors with

$$\frac{c_1}{w_1} \geq \frac{c_2}{w_2} \geq \cdots \geq \frac{c_n}{w_n}$$

and let

$$k = \min\left\{ j \in \{1, \ldots, n\} : \sum_{i=1}^{j} w_i > W \right\}.$$

Show that an optimum solution for the Fractional Knapsack problem is given by

$$x_j = 1 \quad \text{for } j = 1, \ldots, k-1,$$
$$x_j = \frac{W - \sum_{i=1}^{k-1} w_i}{w_k} \quad \text{for } j = k, \text{ and}$$
$$x_j = 0 \quad \text{for } j = k+1, \ldots, n.$$

Exercise 3 (Greedy Knapsack)

Construct an instance for which the approximation-guarantee 1/2 of the Greedy algorithm for Knapsack is achieved.

Exercise 4 (Fractions of Random Variables)

Let $X_1, \ldots, X_n$ be arbitrary positive (not necessarily independent) but identically distributed random variables. Show that for all $i \in \{1, \ldots, n\}$ we have

$$\mathbb{E}\left[\frac{X_i}{\sum_{j=1}^{n} X_j}\right] = \frac{1}{n}.$$