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Sheet 07

Average-Case Analysis

Exercise 1 (To Be or Not To Be)

A jar begins with one amoeba. Every minute, every amoeba turns into 0, 1, 2, or 3 amoebas with a probability of 1/4 for each case (dies, does nothing, splits into two, or splits into three). What is the probability that the amoeba population eventually dies out?

Exercise 2 (Job Arrivals)

In a computer n jobs arrive. Each job is either *long*, with probability $p \geq 0$, or *short*, independently. For $n \to \infty$, what is the probability that the number of long jobs in the system is even?

Exercise 3 (Spam Arrivals)

Suppose that the number of your incoming emails is according to a Poisson process with some rate $\lambda > 0$. (A *Poisson process* $(N_t)_{t \geq 0}$ with rate $\lambda > 0$ has the properties: $N_t \in \mathbb{N}$ for all $t \geq 0$ and $N_0 = 0$, $(N_t)_{t \geq 0}$ has independent increments, and $\Pr[N_{t+s} - N_s = n] = e^{-\lambda t}(\lambda t)^n/n!$ for all $s, t \geq 0$.)

Each arriving email is spam with probability $p \geq 0$, independently. Show that the process counting the number of spam mails received is also a Poisson process. What is its rate?