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Algorithms Theory, Assignment 5

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Exercise 5.1 - Disjoint-set forests [Points: 3+2]

Consider the implementation of disjoint sets, where sets are represented by rooted trees as in the lecture.

- Give a sequence of m *makeSet*, *union*, and $findSet$ operations, n of which are *makeSet* operations, that require in total $\Theta(m \ \lg \ n)$ time. You are allowed to use union by rank, but findSet *without* path compression only.
- Give an iterative version of the $findSet$ procedure (with path compression).

Exercise 5.2 - Ackerman Function [Points: 5]

The Ackerman function $A : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ is defined as follows:

$$A(0, j) = j + 1$$
$$A(k, j) = A^{(j+1)}(k-1, j) \qquad \text{for } k \geq 1$$
$$\text{where } A^{i+1}(k, j) := A(k, A^{i}(k, j)) \text{ and } A^{1}(k, j) = A(k, j)$$

Prove that $A(k + 1, j) \geq A(k, j)$ for any $k, j \geq 0$.

Exercise 5.3 - Greedy algorithms [Points: 2+1+2]

Suppose that we have a set of activities to schedule among a large number of lecture halls. We wish to schedule all the activities using as few lecture halls as possible.

- Describe a greedy algorithm to determine which activity should use which lecture hall.
- Why is your algorithm greedy?
- Show the optimal substructure of this problem.

Exercise 5.4 - Greedy algorithms [Points: 5]

Let $G = (V, E, w)$ be a complete bipartite graph, whit vertexes $V = V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$, $|V_1| = |V_2|$, edges E and a weight function $w : E \rightarrow \mathbb{R}^{+}$. Design a Greedy algorithm which computes a minimal perfect matching from G. When possible, with minimal weights.

Hint: A *minimal perfect matching* is a minimum size edge cover $M \subseteq E$ whit minimun weights, such that for each vertex $v \in V$, v is incident to exactly one edge in M.