Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Dr. Alexander Souza

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Sheet 01

# Combinatorial Optimization

## Exercise 1 (Running Time)

Suppose we are given a graph $G = (V, E)$ on $n$ vertices and $m$ edges with a weight function $w : E \to \mathbb{N}$, where we assume that this function is given as a vector $w = (w(e_1), \ldots, w(e_m))$. We are further given an algorithm that solves a certain problem and takes running time proportional to $n^2 W$, where $W = \sum_{e \in E} w(e)$. Is this a polynomial time algorithm?

## Exercise 2 (Factional Job Assignment)

Consider the following job assignment problem: Given $m$ identical machines and $n$ jobs with processing times $p_j$ and sets $S_j \subseteq \{1, \ldots, m\}$, for $j = 1, \ldots, n$, distribute (fractionally) the computation of the jobs onto the machines such that job $j$ runs only on machines from $S_j$ and the maximal completion time of all machines is minimized. More formally,

$$\begin{array}{rll} \text{minimize} & \displaystyle\max_{i=1,\ldots,m} \sum_{j=1}^{n} x_{ij} & \\ \text{subject to} & \displaystyle\sum_{i \in S_j} x_{ij} = p_j & j = 1, \ldots, n \\ & x_{ij} \geq 0 & i = 1, \ldots, m, \; j = 1, \ldots, n. \end{array}$$

This formulation is almost an LP, except that "max" appears in the objective function. Formulate the problem as an LP.

## Exercise 3 (Guest Shuffle)

Suppose you are organizing a dinner and lay $n$ tables. You invite $m$ families to join the dinner and family $i$ has $a_i$ members. Furthermore table $j$ has $b_j$ seats. In order to boost the inter-family-communication you want to make sure that no two members of the same family are at the same table (if this is possible). Formulate this seating arrangement problem as a maximum flow problem.

## Programming 4 (First Steps with CPLEX)

Download ILOG CPLEX from our website. Follow the instructions for installation provided there. Also download and read the tutorial.