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Sheet 02

Combinatorial Optimization

Exercise 1 (Multi-Source Multi-Sink)

The Maximum Flow problem can be generalized to multiple sources $s_1, \ldots, s_p$ and sinks $t_1, \ldots, t_q$. The value of a flow f is then defined as $\text{val}(f) = \sum_{1 \leq i \leq p} \text{bal}_f(s_i)$. Show that the Multi-Source Multi-Sink Maximum Flow problem can be reduced to the ordinary Maximum Flow problem.

Exercise 2 (Capacity Constraints)

In the lecture we considered flow problems that have capacity constraints on the edges. Another possibility is to consider capacities on the vertices or capacities on both, vertices and edges. Show that these three types of constraints are equivalent in the sense that an instance with one type of constraints can be transformed into an instance with another type of constraints and the instants' sizes only differ by a constant factor.

Exercise 3 (Fractional Constrained Bin Packing)

A company has to store n items. Each item has a certain size and specification. For instance, it is an animal or refrigerated good, etc. The company decides to rent containers from a service provider. The provider offers m different types of containers. Each container type has an associated cost, a size, and a specification (e.g., can store living or refrigerated goods, etc.). An item can only be stored in a container that meets its specification. Further, the provider rents out containers partially. That means, if a container is not completely filled its cost is (linearly) scaled with the ratio of content size to container size.

The company likes to rent containers such that each item is appropriately stored (i.e., meets its specification and no container is filled above its size) and the rental cost is minimized. Note, that multiple containers of the same type are allowed.

Give an LP formulation of this Fractional Constrained Bin Packing problem. Further, give a Greedy algorithm and show that it solves the problem optimally.

Programming 4 (Modeling with CPLEX)

Give an OPL model of the Fractional Constrained Bin Packing problem in ILOG CPLEX.