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Sheet 07

Combinatorial Optimization

Exercise 1 (Online Scheduling)

Consider an *online* scheduling problem for m identical machines where jobs arrive *over time*: The existence of a job is unknown until a certain release date, at which point the processing requirement for that job becomes known. Denote the release date of job j by r_j and its processing requirement by p_j. The goal is to minimize the makespan.
By contrast, in the corresponding *offline* version, all release dates and processing requirements are known at the outset.
An algorithm for an online problem is said to be *c-competitive* if its solution value is always within c times the offline optimal value.
Assume that we are given a c-approximation algorithm for the offline problem. Show that this implies a $2 \cdot c$-competitive algorithm for the online problem.
Hint. Let J_0 be the set of jobs released at time $t_0 = 0$. Apply the approximation algorithm to schedule J_0, finishing at time t_1. Let J_1 be the set of jobs released in the time interval $(t_0, t_1]$. Again, apply the approximation algorithm to schedule J_1, finishing at time t_2. Continue.

Exercise 2 (Bin Packing Lower Bounds)

Give examples that establish lower bounds for the approximation factor of

(a) 5/3 for First Fit

(b) 3/2 for First Fit Decreasing

for Bin Packing.

Exercise 3 (Next Fit with Bounded Sizes)

Let $0 < \gamma < 1$. Let $I = \{1, \dots, n\}$ be an instance of Bin Packing with $s_i < \gamma$ for $i \in I$. Denote the number of bins used by Next Fit on instance I by $\textsc{nf}(I)$.
Show that

$$\textsc{nf}(I) \le \left\lceil \frac{s(I)}{1-\gamma} \right\rceil \le \left\lceil \frac{\textsc{opt}(I)}{1-\gamma} \right\rceil,$$

where $\textsc{opt}(I)$ denotes the optimal number of bins for the instance I.