Locality of weak and not-so-weak coloring

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Hardness of minimal symmetry breaking in distributed computing arXiv:1811.01643

Locality of not-so-weak coloring arXiv:1904.05627
General Topic
LOCAL model

• Entities = nodes
• Communication links = edges
• Input graph = communication graph
LOCAL model

- Each node has a unique identifier from 1 to poly(n)
- No bounds on the computational power of the entities
- No bounds on the bandwidth
LOCAL model

• Round 0
LOCAL model

• Round 1
LOCAL model

• Round 2
LOCAL model

• After $t$ rounds: knowledge of the graph up to distance $t$
• Focus on locality
Weak 2-Coloring

2-coloring where each node has at least a neighbor of different color
Distributed Complexity of Weak 2-Coloring

- $\Theta(\log^* \Delta)$ in odd-degree graphs [Naor and Stockmeyer 1995] [Brandt 2019]
- $O(\log^* n)$ on general graphs
- $\Omega(\log^* n)$ on cycles [Reduction from 3-coloring]
- $\Omega(\log \log^* n)$ on regular trees [Naor and Stockmeyer 1995] [Chang and Pettie 2017]
The $\Omega(\log \log^* n)$ lower bound

- Naor & Stockmeyer proved that any constant time algorithm for LCLs can be transformed to an order invariant algorithm.
- On even regular trees, weak 2-coloring cannot be solved by an order invariant algorithm.
- Chang and Pettie lifted the gap up to $\Omega(\log \log^* n)$.
- Both proofs use Ramsey theory.
- Ramsey gives a lower bound on volume, not distance.
Lower bound on cycles
Lower bound on cycles

$\Omega(\log^* n)$
Lower bound on cycles
Lower bound on trees
Lower bound on trees

$\Omega(\log^* n)$ nodes
Lower bound on trees

Ω(log* n) nodes
Ω(log log* n) distance
Complexity in even degree regular graphs

• Lower bound of $\Omega(\log \log^* n)$ distance and $\Omega(\log^* n)$ volume

• Upper bound of $O(\log^* n)$ distance

• Is a volume of $O(\log^* n)$ nodes enough?

• Or do we need to see at distance $\Omega(\log^* n)$?

Is it easier to solve weak 2-coloring if we have many neighbors?
Our results

Weak 2-coloring requires $\Omega(\log^* n)$ time in even-regular trees:

• For any constant even $\Delta$

• Even if we allow randomization

• Even if identifiers are exactly in $\{1, \ldots, n\}$

Also, weak 2-coloring is the easiest possible non constant time "homogeneous LCL" problem
Speedup Simulation Technique

• Given:
  • an algorithm $A_0$ that solves problem $P_0$ in $T$ rounds,

• We construct:
  • an algorithm $A_1$ that solves problem $P_1$ in $T-1$ rounds,
  • an algorithm $A_2$ that solves problem $P_2$ in $T-2$ rounds,
  • an algorithm $A_3$ that solves problem $P_3$ in $T-3$ rounds,
  • ...
  • an algorithm $A_T$ that solves problem $P_T$ in 0 rounds.

• We prove that $P_T$ cannot be solved in 0 rounds.
Speedup for Weak 2-Coloring

- Given an algorithm $A$ that solves weak $c$ coloring in $T$ rounds, we construct an algorithm $A'$ that solves "special" weak $2^{2c}$ edge coloring in $T-1$ rounds.

- Given an algorithm $A$ that solves "special" weak $c$ edge coloring in $T$ rounds, we construct an algorithm $A'$ that solves weak $2^{4c}$ coloring in $T$ rounds.
## Beyond Weak 2-Coloring

### Weak 2-coloring

- **2-color** the nodes such that each node has at least 1 neighbor of different color

### 2-Partial 2-Coloring

- **2-color** the nodes such that each node has at least 2 neighbors of different color
Our results

• **2-partial 2-coloring** requires:
  • $\Omega(\log n)$ for any constant $\Delta \geq 2$

• **$k$-partial 3-coloring** requires:
  • $\Omega(\log n)$ for $\Delta = k$
  • $O(\log^* n)$ for $\Delta \gg k$
Conclusions

• **Weak 2-Coloring** requires $\Theta(\log^* n)$ time on $\Delta$ regular trees

• Requiring **2 neighbors** of different color, instead of just 1, makes the problem much harder, $\Omega(\log n)$, even if $\Delta = 1000$

• Open problem:
  
  • **3**-partial **3**-coloring on **3**-regular graphs is $\Omega(\log n)$ (it is $\Delta$-coloring)
  
  • **3**-partial **3**-coloring on **5**-regular graphs is $O(\log^* n)$
  
  • What is the complexity of **3**-partial **3**-coloring on **4**-regular graphs?

Thank you!