Albert-Ludwigs-Universität
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June 20, 2012

Network Algorithms, Summer Term 2012
Problem Set 7

hand in by Wednesday, June 27 , 2012, 12:00

Exercise 1: Greedy Dominating Set

The distributed version of the greedy dominating set (DS) algorithm presented in the lecture computes a $\ln \Delta$-approximation in $O(n)$ rounds.

Construct a graph $G = (V, E)$ for which the approximation ratio is as large as possible, i.e., the size of the computed DS is a factor $\Omega(\log \Delta)$ larger than the optimal DS! Try to find a graph for which Δ is as large as possible!

Exercise 2: Fast Dominating Set

The second algorithm discussed in the lecture only needs $O(\log^2 \Delta \log n)$ rounds to compute an $O(\log \Delta)$-approximation. More precisely, the algorithm requires $O(\log^2 \Delta \log n)$ phases, where each phase (i.e., one iteration of the while loop) consists of a constant number of rounds. Write down the communication steps (node v sends/receives ... to/from ...) for a single phase in detail! How many rounds are required exactly in each phase?

Exercise 3: Vertex Cover Approximation

A related (but simpler) problem is the problem of computing a small vertex cover. A vertex cover of a graph $G = (V, E)$ is a subset $S \subseteq V$ of the nodes of G such that for every edge $\{u, v\} \in E$, at least one of the two nodes is in S, i.e., $\{u, v\} \cap S \neq \emptyset$. The objective of the minimum vertex cover problem is to compute a vertex cover of smallest possible size.

1. Show that in a d-regular graph G, taking $S = V$ gives a 2-approximation for the problem
2. Assume that in graph G, all nodes $v \in V$ have degree $\deg(v) \leq \Delta$. Consider the node set $A := \{u \in V : \deg(u) \geq \Delta/2\}$. How large can A be, compared to the smallest vertex cover of G?
3. Can you use your result from b) to get a distributed approximation algorithm for the minimum vertex cover problem? What are approximation ratio and time complexity of your algorithm?