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Network Algorithms, Summer Term 2012
Problem Set 9

Exercise 1: Counting with Asynchronous Wake-up (Start)

Recall the counting problem in edge-dynamic dynamic networks presented in the lecture. Communication is synchronous, message size arbitrary, and each node has an unique identifier. We want all nodes to learn the number of nodes n.

We assume that the dynamic graph $G = (V, E)$ is 2-interval connected, i.e., for any two subsequent rounds r, $r + 1$, the ("static") graph $(V, E(r) \cap E(r + 1))$ is connected.

Now we drop the assumption from the lecture that all nodes wake up at the same time. Instead, some node $u \in V$ wakes up by itself, while all other nodes start executing the respective algorithm when they receive the first message.

1. Show that anything that can be done if a single node u starts the computation and all other nodes arewoken up when they receive the first message, can also be done if nodes can also wake up spontaneously, without receiving a message. Note that nodes still wake up upon receiving the first message if they are not awake by that time.

2. Devise an algorithm that receives an input k and lets u decide whether $k \leq n$ or $k > n$ within $O(k)$ rounds. **Hint: Make u wake up all nodes and collect all identifiers assuming that we have less than k nodes. With a little extra time, one will see more than k identifiers if $n > k$.**

3. Use your algorithm as a subroutine for an algorithm that determines n up to a factor 2 in $O(n)$ time. Can n also be determined exactly?