

## Network Algorithms, Summer Term 2013

### Problem Set 7

#### Exercise 1: Coloring Rings

In Chapter 1, we proved that a ring can be colored with 3 colors in  $\log^* n + O(1)$  rounds. Clearly, a ring can only be (legally) colored with 2 colors if the number of nodes is even.

1. Prove that, even if the nodes in a directed ring know that the number of nodes is even, coloring the ring with 2 colors requires  $\Omega(n)$  rounds!<sup>1</sup>

Since coloring a ring with 2 colors apparently takes a long time, we again resort to the problem of coloring rings using 3 colors.

2. Assume that a *maximal independent set* (MIS) has already been constructed on the ring, i.e., each node knows whether it is in the independent set or not. Assuming that nodes have different identifiers, give an algorithm to color the ring with 3 colors in this scenario! What is the time complexity of your algorithm? Deduce from this a lower bound for computing a MIS!

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<sup>1</sup>As in the lecture, the message size and local computations are unbounded and all nodes have unique identifiers from 1 to  $n$ .