

Network Algorithms, Summer Term 2013

Problem Set 9

Counting with Asynchronous Wake-up (Start)

Recall the counting problem in edge-dynamic dynamic networks presented in the lecture. Communication is synchronous, message size arbitrary, and each node has a unique identifier. We want all nodes to learn the number of nodes n .

We assume that the dynamic graph $G = (V, E)$ is 2-interval connected, i.e., for any two subsequent rounds $r, r + 1$, the (“static”) graph $(V, E(r) \cap E(r + 1))$ is connected.

Now we drop the assumption from the lecture that all nodes wake up at the same time. Instead, some node $u \in V$ wakes up by itself, while all other nodes start executing the respective algorithm when they receive the first message.

1. Show that anything that can be done if a single node u starts the computation and all other nodes are woken up when they receive the first message, can also be done if nodes can also wake up spontaneously, without receiving a message. Note that nodes still wake up upon receiving the first message if they are not awake by that time.
2. Devise an algorithm that receives an input k and lets u decide whether $k \leq n$ or $k > n$ within $O(k)$ rounds. **Hint: Make u wake up all nodes and collect all identifiers assuming that we have less than k nodes. With a little extra time, one will see more than k identifiers if $n > k$.**
3. Use your algorithm as a subroutine for an algorithm that determines n up to a factor 2 in $O(n)$ time. Can n also be determined exactly?