

## Network Algorithms, Summer Term 2013

### Problem Set 7 – Sample Solution

#### Exercise 1: Coloring Rings

1. Let  $n \geq 4$  be even, and  $r = n/2 - 2$ . Consider the  $r$ -neighborhood graph  $\mathcal{N}_r(R_n)$  of the ring  $R_n$  with  $n$  nodes. Note that for  $r = n/2 - 2$  the  $r$ -neighborhood of a node contains all but three identifiers, ordered according to their occurrence.

The ring can be colored legally with two colors if and only if  $\mathcal{N}_r(R_n)$  is bipartite, i.e., the  $r$ -neighborhood contains no odd cycle. However, there is one of length  $n - 1$ :

$(1, \dots, n - 3), (2, \dots, n - 2), (3, \dots, n - 1), (4, \dots, n - 1, 1), (5, \dots, n - 1, 1, 2), \dots,$   
 $(n - 1, 1, 2, \dots, n - 4), (1, \dots, n - 3).$

Thus no coloring of the ring with 2 colors is possible in less than  $n/2 - 1$  rounds.

2. Each node informs its two neighbors whether it is in the MIS or not and additionally sends its identifier. If node  $v$  is in the MIS, it sets its color to 1. If  $v$  is not in the MIS but both of its neighbors are, then  $v$  sets its color to 2. If  $v$  has a neighbor  $w$  not in the MIS,  $v$  chooses color 2 if its identifier is larger than  $w$ 's identifier, otherwise  $v$  chooses the color 3.

The algorithm only needs one communication round. Correctness follows from the fact that either a node  $v$  is in the MIS or at least one of its neighbors is. Thus, a MIS can at best be computed one round faster than a 3-coloring, which implies that computing a MIS costs at least  $(\log^* n)/2 - 2$  rounds.