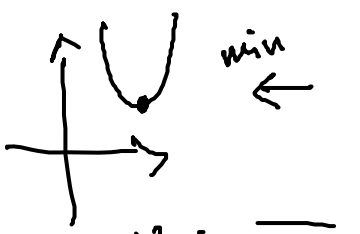
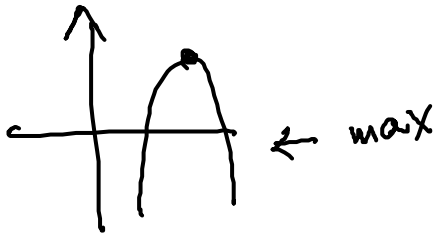
$f$, $g_i = 0$ $\Rightarrow$ $L(x, \lambda) = f(x) + \sum_{i=1}^{n} \lambda_i g_i(x)$ $\quad L(x^*, \lambda^*) = 0$ $\quad x^*$ - st. punkt. $f$




min + max.

$\nabla f(x^*) = 0$

1) Zulässig: $g_i(x^*) = 0 \quad \forall\, i = \overline{1,n}$

2) keine (Abstiegsrichtung): $\forall s \quad s^T \nabla f(x^*) \overset{!}{=} 0 \;\wedge\; s^T \nabla g(x^*) = 0$

1) min
2) max
3) $(\lambda_1^*, \lambda_2^* \dots \lambda_n^*)^T$

$\overset{!}{=} 0$ ← general case

if $\exists\, \lambda_i^*$ $L(x^*, \lambda^*) = 0$ $\Rightarrow$ $x^*$ - st. p. $\nexists$ abstiegsrichtung. ← reicht für $x^*$

$\nexists$ aufstiegsrichtung.

$-f(x) = \sum_{i=1}^{n} \lambda_i g_i(x)$ $\quad s$

$0 \overset{!}{=} -s^T f(x) = \sum_{i=1}^{n} \lambda_i s^T g_i(x) = 0$ ↯

$f_{max} = xyz$ — max / min

$g = 2(xy + yz + xz) - A = 0$

$L = xyz + \lambda[2(xy + yz + xz) - A]$

$\frac{\partial L}{\partial x} = yz + 2\lambda[y + z] = 0 \quad (1)$

$\frac{\partial L}{\partial y} = xz + 2\lambda[x + z] = 0 \quad (2)$

$\frac{\partial L}{\partial z} = xy + 2\lambda[x + y] = 0 \quad (3)$

$\frac{\partial L}{\partial \lambda} = 2(xy + yz + xz) - A = 0$

$\frac{1}{2}\frac{\partial L}{\partial \lambda} = xy + yz + xz - \frac{A}{2} = 0$

$\lambda = -\frac{yz}{2(y+z)}$

$y + z \neq 0$

$z \neq 0$

$x \neq 0$ ←

$xz - 2\lambda \cdot \frac{yz}{2(y+z)}(x+z) = 0$

$\frac{xyz + xz^2 - yzx - yz^2}{y+z} = \frac{(x-y)z^2}{y+z} = 0$ — $z'$

$(x-y)z^2 = 0$

$x = y$

$xy - 2\lambda \cdot \frac{yz}{2(y+z)}(x+y) = 0$

$\frac{xy^2 + xyz - xyz - y^2z}{(y+z)} = \frac{y^2(x-z)}{y+z} = 0$ — (3)′

$y^2(x-z) = 0$

$\begin{cases} x^2(x-z) = 0 & x = z \\ x^2 + 2xz - A = 0 \end{cases}$

$3x^2 = \frac{A}{2}$ $\quad x = y = z = \pm\sqrt[2]{\frac{A}{6}}$ ←

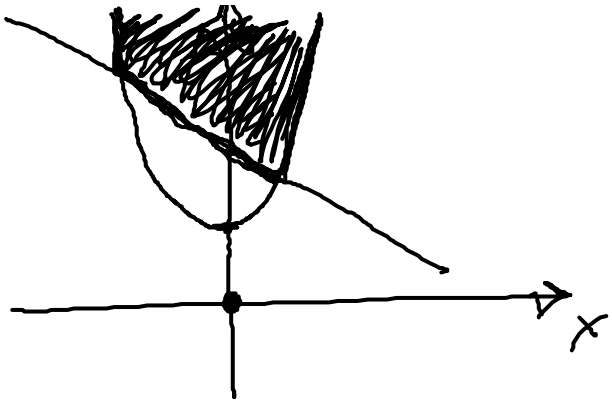
$\sqrt{\frac{A}{6}} = x = y = z$ ← max

— // — ← min

$x^2+2y^2 \to \min$

$g_1$: $x^2-y+2 \le 0$ $\Leftarrow$ $\le$ $=0$

$g_2$: $4-x-y \le 0$ $\Leftarrow$




$$L = x^2+2y^2+\lambda_1(x^2-y+2)+\lambda_2(4-x-y)$$

KKT Bedi...

$$0=\frac{\partial L}{\partial x} = 2x+2\lambda_1 x-\lambda_2 = 0$$

$$0=\frac{\partial L}{\partial y} = 4y-\lambda_1-\lambda_2 = 0$$

$g_1 \le 0$ $\quad x^2-y+2 \le 0$ •

$g_2 \le 0$ $\quad 4-x-y \le 0$ •

$\lambda_1 \cdot g_1 = 0$ $\quad \lambda_1(x^2-y+2)=0$ •

$\lambda_2 \cdot g_2 = 0$ $\quad \lambda_2(4-x-y)=0$ •

$\lambda_i \ge 0$ $\quad \lambda_1, \lambda_2 \ge 0$

1) $\lambda_1=\lambda_2=0$

$2x=0$ $\quad (0,0)$

$4y=0$

$0-0+2 \le 0$ ↯

$4-0-0 \le 0$ ↯

2) $\lambda_1=0, \lambda_2>0$

$4-x-y=0$

$2x-\lambda_2=0$

$4y-\lambda_2=0$

$\lambda_2=\frac{16}{3}$ $\quad \lambda_1=0$

$x=\frac{4}{3}$ $\quad y=\frac{8}{3}$

$\frac{16}{9}-\frac{8}{3}+2 \le 0$

$\frac{16-24+18}{9}$ ↯

3) $\lambda_1>0, \lambda_2=0$

$2x+2\lambda_1 x=0$ $\Leftarrow$

$4y-\lambda_1=0 \to \lambda_1=8$

$x^2-y+2=0$

$\left[\begin{array}{l} x=0 \\ 1+\lambda_1=0 \Leftarrow \text{↯} \end{array}\right.$

$x=0$

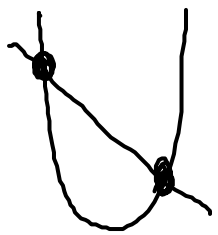
$-y+2=0$ $\quad y=2$

$(0,2)$

$4-0-2 \le 0$ ↯

4) $\lambda_1, \lambda_2 \geq 0$

$$\begin{cases} x^2 - y + 2 = 0 \\ 4 - x - y = 0 \end{cases}$$



$2x + 2\lambda_1 x - \lambda_2 = 0$

$x = 1 \quad y = 3$ → ~~kein~~ Solution

$x = -2 \quad y = 6$ → $-4 - 2\lambda_1 - \lambda_2 = 0 \Rightarrow -4 = 2\lambda_1 + \lambda_2 \geq 0$ ↯

---

3) $\max c^T x$

$Ax \leq b$

$x_i \geq 0$

$\min f \rightarrow -1 * \max -f$

$g_i \geq d_i \quad -1 *$

$g_j' = d_j' \quad \geq \wedge \leq \quad g \pm d \geq \leq = 0$

$g_k'' \leq d_k''$

$x_i \geq 0 \quad x_j' \text{ - beliebig} \quad x_k'' \leq 0$

$-y_k'' = x_k'' \quad y_k'' \geq 0$

$x_j' = \alpha_j - \beta_j$

$x_j \geq 0$

$\beta_j \geq 0$

$\min 2w + 7x + 11y + 15z$

$w + x - 12y + 3z \geq 0$

- $4x + y - z = 1$

$$\begin{cases} 4x + y - z \geq 1 \\ 4x + y - z \leq 1 \end{cases}$$

$-x + 7y + 3z \leq 8$

$w \geq 0$

$x \geq 0$

$y, z \in \mathbb{R}$

$$\begin{cases} y = y_1 - y_2 \\ z = z_1 - z_2 \end{cases}$$

$-w - x + 12y - 3z \leq 0$

$-4x - y + z + 1 \leq 0$

$4x + y - z - 1 \leq 0$

$-x + 7y + 3z - 8 \leq 0$

$\max -2w - 7x - 11y_1 + 11y_2 - 15z_1 + 15z_2$

$-w - x + 12y_1 - 12y_2 - 3z_1 + 3z_2 \leq 0$

$-4x - y_1 + y_2 + z_1 - z_2 + 1 \leq 0$

$4x + y_1 - y_2 - z_1 + z_2 - 1 \leq 0$

$-x + 7y_1 - 7y_2 + 3z_1 - 3z_2 - 8 \leq 0$

$w \geq 0$

$x \geq 0$

$y_1, y_2 \geq 0$

$z_1, z_2 \geq 0$