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Institut für Informatik
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Network Algorithms, Summer Term 2014
Problem Set 2

hand in by Thursday, June 5, 2014

Exercise 1: Almost Anonymous Leader Election

For this exercise, assume that nodes are almost anonymous and have only two different identifiers 0 and 1. We consider deterministic, synchronous, uniform leader election algorithms on the ring. You can assume that nodes can distinguish between their two neighbors, i.e., when a node v receives a message, v knows which neighbor has sent the message (note that nodes may not know a consistent clockwise or counterclockwise orientation of the ring!)

1. Assume that exactly 3 nodes have identifier 1 (we assume that the algorithm knows this!). Give a uniform, deterministic algorithm that elects a leader whenever it is possible. For which values of n is it always possible to elect a leader?

2. Now, assume that the algorithm does not know anything about the number of nodes with identifier 0 or 1 and neither anything about n. Prove that for all executions (i.e., for all n and all assignments of IDs), no uniform synchronous algorithm can elect a leader.

Exercise 2: Leader Election in Trees

We now assume that the communication graph is an anonymous tree. Note that we assume that the tree is undirected and there is no distinguished root node. Assume that each node knows its degree and that each node can distinguish all its neighbors. Show that if n is odd, it is always possible to elect a leader. Give a deterministic, asynchronous algorithm and analyze its time and message complexity.

Exercise 3: License to Match

In preparation of a highly dangerous mission, the participating agents of the gargantuan Liechtensteinian secret service (LSS) need to work in pairs of two for safety reasons. All members in the LSS are organized in a tree hierarchy. Communication is only possible via the official channel: an agent has a secure phone line to his direct superior and a secure phone line to each of his direct subordinates. Initially, each agent knows whether or not he is taking part in this mission. The goal is for each agent to find a partner.

1. Devise an algorithm that will match up a participating agent with another participating agent given the constrained communication scenario. A "match" consists of an agent knowing the identity of his partner and the path in the hierarchy connecting them. Assume that there is an even number of participating agents so that each one is guaranteed a partner. Furthermore, observe that[1] the phone links connecting two paired-up agents need to remain open at all times after the mission starts. Therefore, you cannot use the same link (i.e., an edge) twice when connecting agents with their partners (i.e., the paths need to be edge-disjoint).

2. What are the time and message (i.e., "phone call") complexities of your algorithm?

[1] in the case of an emergency where they lose contact