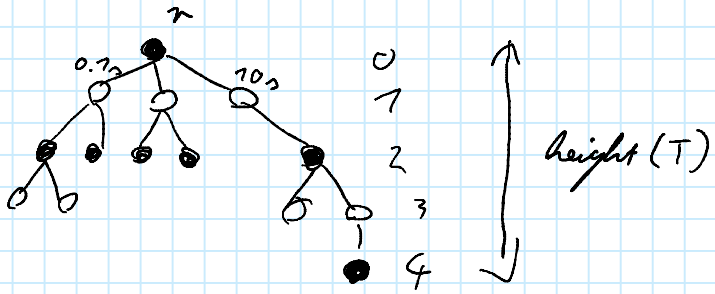
Coloring Trees

$\chi(T) \leq 2$



Slow Tree Coloring

root: color itself with 0, send color to children

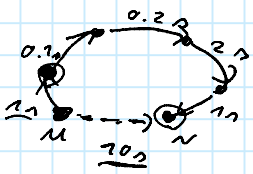
each node v:
- wait until receiving msg c_p from parent
- color itself with $c_v = 1 - c_p = 1 + c_p \bmod 2$
- send c_v to children

time compl.: $O(\text{height}(T))$

msg compl.: $n - 1$

Def.: asynchronous distr. algorithm

- event driven
- no global clock
- no bound on message delays (but finite)



time compl.

time units in worst case execution

.... ?

normalize longest delay as one time unit

$\log^* n$

$\forall x \leq 2: \quad \log^* x := 1$

$\forall x > 2: \quad \log^* x := 1 + \log^*(\log x)$

$10^{80} \sim 2^{240} \leq 2^{256}$

$\log^* 10^{80} < \log^* 2^{256}$

$\log^* 10^{80} = 5$

$2^{256} \to 256 \to 8 \to 3 \to 1.6\ldots$ (1 2 3 5 5)

6-Color

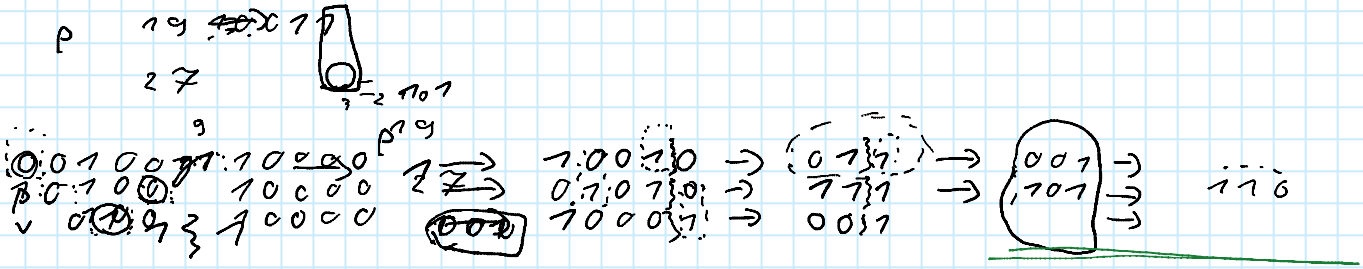
start with initial valid coloring (e.g. ID's!)

root gets $c_r = 0$

$\forall$ nodes v: (synchronously in parallel)

- send c_v to children
- repeat
  - receive c_p from parent p
  - interpret c_p & c_v as bit strings,
    i.e.: $c_v := c_v(k)\, c_v(k-1) \ldots c_v(1)\, c_v(0)$
  - $i :=$ smallest index where c_v & c_p differ
  - new color $c_v' := 2 \cdot (i)_b + c_v(i)$
    $= (i)_b \square c_v(i)$

until all colors are in $\{0, \ldots, 5\}$



Correctness

$c_p \neq c_v$

↓

one iteration

↓

$c'_p \neq c'_v$?

c_p' : i_p $\{$ $c_p(i_p)$

c'_v : i_v $\{$ $c_v'(i_v)$

1st case: $i_v \neq i_p$ $\Rightarrow$ ✓

2nd case: $i_v = i_p$ $\Rightarrow$ by definition of i_v

we have that $c_v(i_v) \oplus c_p(i_v) \overset{i_v = i_p}{=\!=} c_p(i_p)$

$\Rightarrow c'_v \neq c'_p$

time compl.:

c_v highest color in T

$\Rightarrow \forall w: c'_w \leq 2 \cdot \log c_v + 1$

$\Rightarrow$ also $c'_{image} \leq 2 \cdot \log c_v + 1$

$\Rightarrow \ldots \Rightarrow \log^* n + O(1)$

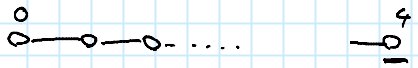
progress

c_v highest color in T

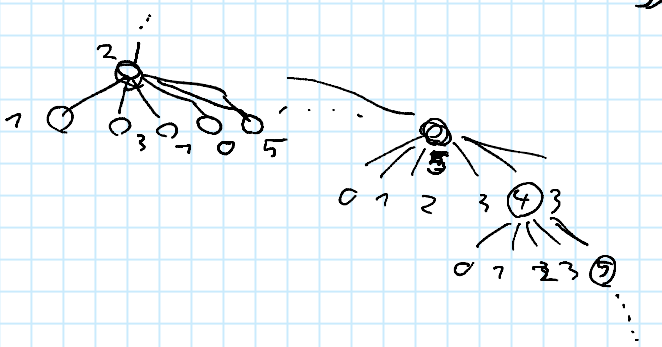
$c_v \geq 8 \Rightarrow \forall w: c_w < \ldots \leq c_v$

$c_v \in \{6, 7\} \Rightarrow \forall w: i_w \leq 2 \Rightarrow c_w \leq 5$

"until all colors ≤ 5" → local? how do nodes know?



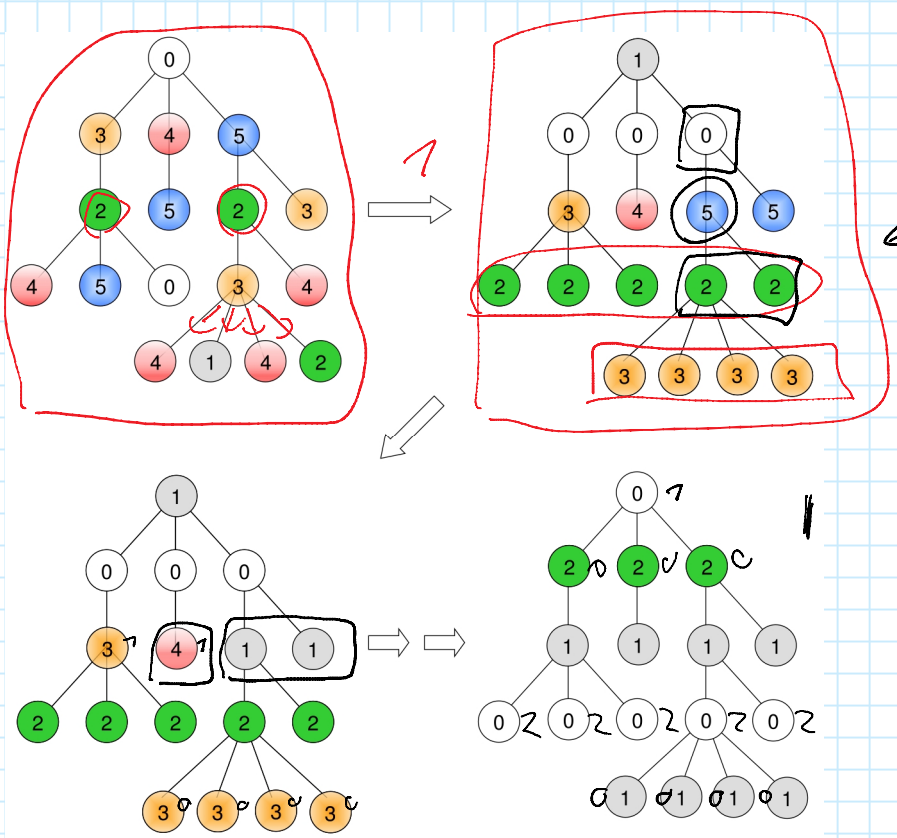
$\Rightarrow$ 2 colors not possible $\rightarrow$ Time $\Omega(\text{height}(T))$



Shift Down

send c_v to children

upon receive c_p change color to c_p



6 to 3

Run "6-color" for $\log^* n$ rounds

Perform Shift Down once

$\forall v$: for $j = 5, 4, 3$ do

if $(c_v = j)$ choose new $c_v \in \{0, 1, 2\}$

next j