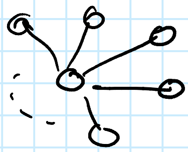
Some remarks

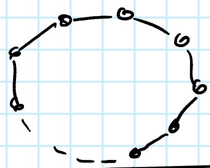
- $\exists$ instances with comp. ratio is $\Omega\left(\frac{\log |S|}{\log\log |S|}\right)$
- analysis can be extended to a dyn. case
- congestion:



T: star

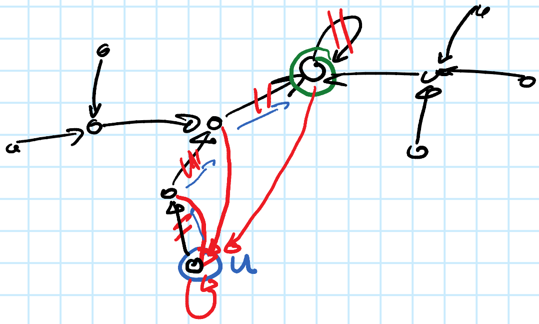
- choice of spanning tree is important
  $\Longrightarrow$ good stretch

good approx



Ivy Protocol

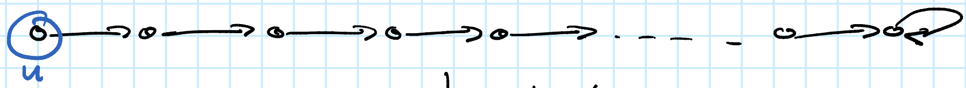
Assumption $G = K_n$



Sequential Case

"non-concurrent" : requests happen after each other

↳ cost per request is small



↓ cost: linear in n

Use amortized analysis

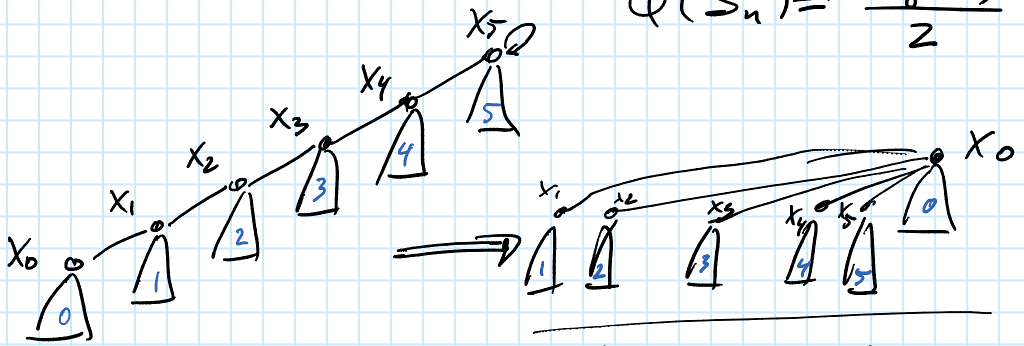
Theorem: If the initial tree is a star, then the average $\underbrace{\text{cost}}_{\text{\#hops to the object}}$ per request is $\leq \log_2 n$

Proof: node u : $s(u)$: #nodes in subtree of u (incl. node u)

$$\Phi(T) := \sum_{u \in V} \frac{\log s(u)}{2}$$

$$\phi(T) \geq \frac{\log(n)}{2} \qquad \text{star } S_n$$

$$\phi(S_n) = \frac{\log(n)}{2}$$



T_i : tree after request i

Request i :

path to object is $\underbrace{x_0, x_1, \ldots, x_{k_i}}$

length k_i = cost of req. i

amortized cost of req. i :

$$a_i := k_i + \phi(T_i) - \phi(T_{i-1})$$

$$\sum_{i=1}^{N} a_i = \sum_{i=1}^{N} \left(k_i + \phi(T_i) - \phi(T_{i-1})\right)$$

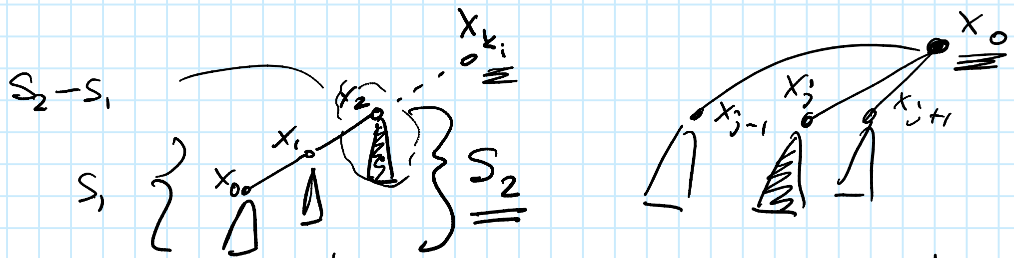
$$= \underbrace{\phi(T_N) - \phi(T_0)}_{\geq 0} + \underbrace{\sum_{i=1}^{N} k_i}_{\text{total cost}}$$

$$\frac{1}{N}\sum a_i \geq \underbrace{\frac{1}{N}\sum k_i}_{\text{avg. cost}} \qquad \boxed{a_i \leq \log_2 n}$$

$$\phi(T) := \sum_{u \in V} \frac{\log s(u)}{2}$$

$$a_i = k_i - \phi(T_{i-1}) + \phi(T_i)$$

$S_j := s(x_j)$ before request i



$$a_i = k_i - \left(\sum_{j=0}^{k_i} \frac{1}{2} \log s_j\right) + \left(\frac{1}{2}\log s_{k_i} + \sum_{j=1}^{k_i} \frac{1}{2}\log(s_j - s_{j-1})\right)$$

$$= k_i - \left(\sum_{j=0}^{k_i-1} \frac{1}{2} \log s_j\right) + \left(\sum_{j=0}^{k_i-1} \frac{1}{2}\log(s_{j+1} - s_j)\right)$$

$$= k_i + \frac{1}{2} \cdot \sum_{j=0}^{k_i-1} \log\left(\frac{s_{j+1} - s_j}{s_j}\right)$$

$\alpha_j - 1$

$$\alpha_j := \frac{s_{j+1}}{s_j} \qquad \alpha_j > 1$$

$$a_i = k_i + \frac{1}{2}\sum_{j=0}^{k_i-1} \log(\alpha_j - 1) = \sum_{j=0}^{k_i-1}\left(1 + \frac{1}{2}\log(\alpha_j - 1)\right)$$

use that for $\alpha > 1$: $1 + \frac{1}{2}\log(\alpha - 1) \leq \log \alpha$

$$a_i \leq \sum_{j=0}^{k_i-1} \log(\alpha_j) = \sum_{j=0}^{k_i-1} \log \frac{s_{j+1}}{s_j} = \sum_{j=0}^{k_i-1} (\log s_{j+1} - \log s_j)$$

$$= \log \underbrace{s_{k_i}}_{=n} - \underbrace{\log s_0}_{\geq 0} \leq \underline{\log n}$$

$\underline{\alpha > 1:}$ $\quad \underline{\underline{1 + \frac{1}{2}\log(\alpha - 1)}} \leq \underline{\underline{\log\alpha}}$

$$\log\alpha = \frac{1}{2}\log\alpha^2$$

$$(\alpha-2)^2 = \alpha^2 - 4\alpha + 4 \geq 0$$

$$\Downarrow$$

$$\alpha^2 \geq 4\alpha - 4$$

$$\geq \frac{1}{2}\log(4(\alpha-1))$$

$$= \frac{1}{2}\left(2 + \log(\alpha-1)\right)$$

$$= 1 + \frac{1}{2}\log(\alpha-1) \quad \checkmark$$