

Network Algorithms, Summer Term 2015 Problem Set 3

hand in by Tuesday, May 12, 2015

Exercise 1: Leader Election in an “Almost Anonymous” Ring

For this exercise, assume that nodes are almost anonymous and have only two different identifiers 0 and 1. We consider deterministic, synchronous, uniform leader election algorithms on the ring. You can assume that nodes can distinguish between their two neighbors, i.e., when a node v receives a message, v knows which neighbor has sent the message (note that nodes may not know a consistent clockwise or counterclockwise orientation of the ring!)

- a) Is deterministic leader election possible in a synchronous ring in which all but one processors have the same identifier? Either give an algorithm or prove an impossibility result.
- b) Assume that exactly 3 nodes have identifier 1 (we assume that the algorithm knows this!). Give a uniform, deterministic algorithm that elects a leader whenever it is possible. For which values of n is it always possible to elect a leader?
- c) Now, assume that the algorithm does not know anything about the number of nodes with identifier 0 or 1 and neither anything about n . Prove that for all executions (i.e., for all n and all assignments of IDs), no uniform synchronous algorithm can elect a leader.

Exercise 2: Leader Election in Trees

We now assume that the communication graph is an anonymous tree. Note that we assume that the tree is undirected and there is no distinguished root node. Assume that each node knows its degree and that each node can distinguish all its neighbors. Show that if n is odd, it is always possible to elect a leader. Give a deterministic, asynchronous algorithm and analyze its time and message complexity.