Albert-Ludwigs-Universität, Inst. für Informatik
Prof. Dr. Fabian Kuhn
M. Ahmadi, P. Bamberger, H. Ghodselahi, Y. Maus, P. Schneider

Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 3

Hand in (electronically or hard copy) by 12:15 pm, May 29th, 2017

Exercise 1: Regular Expressions (3 points)

Give a regular expression, which defines the language U of well-formed URL's (Uniform Resource Locators) or a meaningful, *non-finite* subset thereof. An example of a well-formed URL $u \in U$ is

$$u = \texttt{https://en.wikipedia.org/wiki/URL}$$

Assume that the alphabet is $\Sigma = A \cup B$ with $A = \{a, b, c, \ldots, y, z\}$ and $B = \{:, /, .\}$ (feel free to extend the alphabet if necessary).

Hint: You can assume that any finite sets, which you require to construct your regular expression, like e.g. the set of valid top level domains $D := \{com, de, org, net, \ldots\} \subseteq \Sigma^*$, are given (briefly describe any set you are using). Your solution does not have to cover all aspects of URL's. Restrict yourself to certain cases if you feel that otherwise your regular expression is becoming too complex.

Exercise 2: Limits of the Pumping Lemma (2+4 points)

Consider the language $L = \{c^m a^n b^n \mid m, n \geq 0\} \cup \{a, b\}^*$ over the alphabet $\Sigma = \{a, b, c\}$.

a) Describe in words (not using the pumping lemma), why L can not be a regular language.

b) Show that the property described in the Pumping Lemma is a necessary condition for regularity but not sufficient for regularity.

Hint: Use L as counter example, i.e., show that it can be 'pumped' (in the sense of the pumping lemma), but is still not regular.

Exercise 3: Applications of the Pumping Lemma (3+3 points)

Show that the following languages over the alphabet $\Sigma = \{a, b\}$ are not regular:

a) $L = \{a^m \mid \text{m is a square number}\}$ *(m being a square number means that $m = n^2$ for some non negative integer n)*

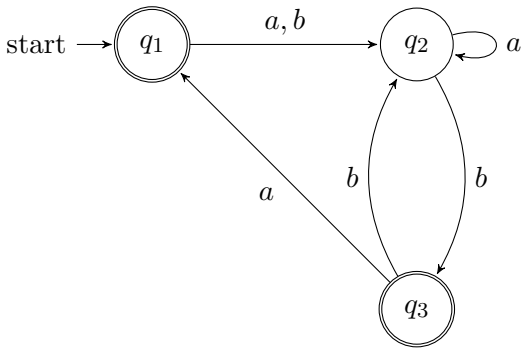
Hint: Use the Pumping Lemma.

b) $L = \{a^m b^n \mid m \neq n\}$

Hint: Have a look at the languages $\{a^n b^n \mid n \in \mathbb{N}\}$ and $a^* b^*$ and use the fact that the class of regular languages is closed under intersection, complement, concatenation and the Kleene star.

Exercise 4: GNFA (5 points)

Consider the following NFA:



Give the regular expression defining the language recognized by this NFA by *stepwise* converting it into an equivalent GNFA with only two nodes.