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# Theoretical Computer Science - Bridging Course
# Summer Term 2017
# Exercise Sheet 4

**Hand in (electronically or hard copy) by 12:15 pm, June 12th, 2017**

## Exercise 1: Context-Free Languages (2+2+1 points)

Give context-free grammars that generate the following languages. The alphabet set is $\Sigma = \{0, 1\}$.

a) $\{w \mid w$ contains at least three ones$\}$

b) $\{w \mid w$ the length of $w$ is odd and its middle symbol is a $0\}$

c) The empty language.

*Note: The empty language is not the language containing only the empty string!*

## Exercise 2: Chomsky Normal Form (2+5 points)

Consider the following context-free grammar (CFG):

$$S \to aSb \mid D$$
$$D \to ccDcc \mid \epsilon$$

a) Which language does this grammar define?

b) Convert this CFG into an equivalent one in Chomsky Normal Form. Give the grammar you obtained after each step of the conversion algorithm.

## Exercise 3: Pushdown Automata (6 points)

Convert the following CFG to an equivalent pushdown automaton. The alphabet is $\Sigma = \{a, +, \times, (, )\}$ and the set of variables $V = \{E, T, F\}$.

$$E \to E + T \mid T$$
$$T \to T \times F \mid F$$
$$F \to (E) \mid a$$

(You already saw this grammar in the lecture with $< expression >$, $< term >$ and $< factor >$ instead of $E, T$ and $F$).

## Exercise 4: Context-Free Languages and Set Operations (3+3 points)

(a) Show that context-free languages are not closed under taking intersections (i.e., the intersection of two context-free languages is not necessarily context free).
*Hint: You can use that the language* $\{a^i b^i c^i | i \geq 0\}$ *is not context-free.*

(b) Show that context-free languages are not closed under taking complements.
*Hint: You can use DeMorgan's law and the fact that the set of context-free languages is closed under performing union operations.*

## Exercise 5: Pumping Lemma for Context-Free Languages (3+3 points)

Use the pumping lemma to show that the following languages over the alphabet $\Sigma = \{a, b\}$ are not context free:

(a) $\{ww \mid w \in \{a, b\}^*\}$

*Hint: Show that the string* $s = a^p b^p a^p b^p$ *with* $p$ *the pumping length cannot be pumped.*

(b) $\{a^n b a^{2n} b a^{3n} \mid n \geq 0\}$

*(Be careful to read the strings correctly: For example* $ab^4$ *is equal to abbbb and not to abababab.)*