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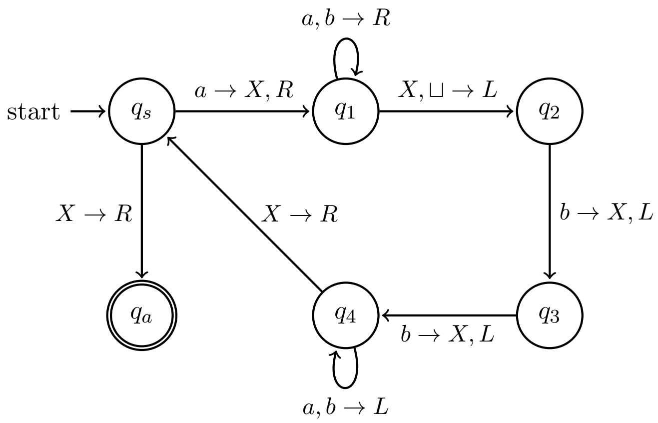
Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 5

Hand in (electronically or hard copy) by 12:15 pm, June 19th, 2017

Exercise 1: Turing Machines Given as State Diagrams (2+3+2 points)

Consider the Turing machine M over the alphabet $\Sigma = \{a, b\}$, which is given via the following state diagram. *Note: The blank symbol* $\sqcup \in \Gamma$ *represents an empty cell on the tape.*



(a) Simulate M with input $s_1 = aabbbb$ on its tape until it halts. Give the configurations that M passes through. You may omit configurations where no symbol is replaced. State whether $s_1 \in L$.

(b) Simulate M with input $s_2 = aabbb$ on its tape until it halts. Give the configurations that M passes through. You may omit configurations where no symbol is replaced. State whether $s_2 \in L$.

(c) Give a description of the language $L(M)$ that M recognizes in the form of a set.

Exercise 2: Formal Definition of Turing Machines (5 points)

Let M be a Turing machine over the alphabet $\Sigma = \{0, 1\}$, with state set $Q = \{q_0, q_1, q_2, q_3, q_4, \overline{q}\}$, starting state q_0, accepting state $\overline{q}$ and transition function δ given via the following table

	0	1	$\sqcup$
q_0	$(q_0, 0, R)$	$(q_0, 1, R)$	$(q_1, \sqcup, L)$
q_1	$(q_2, \sqcup, R)$	$(q_3, \sqcup, R)$	$(\overline{q}, \sqcup, R)$
q_2	$(q_4, 0, L)$	$(q_4, 0, L)$	$(q_4, 0, L)$
q_3	$(q_4, 1, L)$	$(q_4, 1, L)$	$(q_4, 1, L)$
q_4	$(q_4, 1, R)$	$(q_4, 0, R)$	$(q_1, \sqcup, L)$

The above Turing machine accepts every word, i.e., it recognizes the language Σ^*. Describe the behaviour of M on an arbitrary input $w \in \{0,1\}^*$ to explain how the content of the tape after the computation is related to the input word. Do not forget to explain **how** the Turing machine achieves its goal (in particular describe the role of q_2 and q_3) - it is not sufficient (to gain full points) to only name the final result.

Exercise 3: Designing a Turing Machine (2+2+2+2 points)

Let $\Sigma = \{0,1\}$. For a string $s = s_1 s_2 \dots s_n$ with $s_i \in \Sigma$ let $s^R = s_n s_{n-1} \dots s_1$ be the *reversed* string. *Palindromes* are strings s for which $s = s^R$. Then $L = \{sas^R \mid s \in \Sigma^*, a \in \Sigma \cup \{\varepsilon\}\}$ is the language of all palindromes.

(a) Give a state diagram of Turing machine recognizing L.

(b) Give the maximum number of head movements (or a close upper bound) your Turing machine makes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on its tape.

(c) Describe (informally) the behavior of a 2-tape Turing machine which recognizes L and uses significantly fewer head movements on long inputs than your 1-tape Turing machine.

(d) Give the maximum number of head movements (or a close upper bound) your Turing machine makes on any of the two tapes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on the first tape.