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Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 6

Hand in (electronically or hard copy) by 12:15 pm, June 26th, 2017

Exericse 1: Decidability? (3 Points)

Let n be a fixed positive integer, $\Sigma = \{0, 1\}$ a fixed alphabet, M a fixed TM and $w \in \Sigma^*$ a fixed word.

$$L_{\Sigma,M,n,w} := \begin{cases} \{1^n\}, & M \text{ stops on } w \text{ in at most } n \text{ steps} \\ \{0^n\}, & M \text{ stops on } w \text{ after} > n \text{ steps} \\ \emptyset, & M \text{ does not stop on } w. \end{cases}$$

Is $L_{\Sigma,M,n,w}$ decidable?
Remark: For some $a \in \Sigma$, a^n denotes the word which repeats a n times.

Exersive 2: Semi-Decidable vs. Recursively Enumerable (4 Points)

Very often people in computer science use both terms equivalently. The following exercise shows in which way they actually are equivalent. We first recal the definition of both terms.

A language L is *semi-decidable* if there is a Turing machine which accepts every $w \in L$ and does not accept any $w \notin L$ (this means the TM can either reject $w \notin L$ or simply not stop for $w \notin L$.

A language is *recursively enumerable* if there is a Turing machine which eventually outputs every word $w \in L$ and never outputs a word $w \notin L$.

(a) Show that any recursively enumerable language is semi-decidable.

(b) Show that any semi-decidable language is recursively enumberable.

Exercise 3: Halting Problem (3+2+2+1 points)

The *special halting problem* is defined as

$$H_s = \{\langle M \rangle \mid \langle M \rangle \text{ encodes a TM and } M \text{ halts on } \langle M \rangle\}.$$

(a) Show that H_s is undecidable.

Hint: Assume that M is a TM which decides H_s and then construct a TM which halts iff M does not halt. Use this construction to find a contradiction.

(b) Show that the special halting problem is recursively enumerable.

(c) Show that the complement of the special halting problem is not recursively enumerable.

Hint: What can you say about a language L if L and its complement are recursively enumerable? (if you make some observation for this, also prove it)

(d) Let L_1 and L_2 be recursively enumerable languages. Is $L_1 \setminus L_2$ recursively enumerable as well?

Exercise 4 (2+3 points)

(a) Show that every *finite* language is a decidable.

(b) Assume that π is a *fixed* order of the words in Σ^* for which a Turing machine P exists that decides in *finite* time whether $\pi(w) \leq \pi(w')$ for all $w, w' \in \Sigma^*$. Furthermore, assume that for a given language L there is a Turing machine M that enumerates *all* words of a given language L in order $w_1, w_2, w_3, \ldots$ such that $\pi(w_i) \leq \pi(w_j)$ holds for all $i \leq j$. Show that L is decidable.