Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Winter Term 2016
Exercise Sheet 10

Hand in (electronically or hard copy) before your weekly meeting but not later than 23:59, Wednesday, January 18, 2016

Exercise 1: Propositional Logic: Basic Terms (2+2+2+2 points)

Let $\Sigma := \{p, q, r\}$ be a set of atoms. An interpretation $I : \Sigma \to \{T, F\}$ maps every atom to either true or false. Inductively, an interpretation I can be extended to composite formulae φ over Σ (cf. lecture). We write $I \models \varphi$ if φ evaluates to T (true) under I. In case $I \models \varphi$, I is called a *model* for φ.

For each of the following formulae, give *all* interpretations which are models. Make a truth table and/or use logical equivalencies to find all models (document your steps). Which of these formulae are satisfiable, which are unsatisfiable and which are tautologies?

(a) $\varphi_1 = (p \to q) \to r$

(b) $\varphi_2 = (p \land \neg p) \leftrightarrow (q \lor r)$

(c) $\varphi_3 = ((p \to q) \land (q \to r)) \to (p \to r)$

(d) $\varphi_4 = p \lor (p \land q) \not\leftrightarrow p$

Remark: $a \to b :\equiv \neg a \lor b$, $a \leftrightarrow b :\equiv (a \to b) \land (b \to a)$, $a \not\leftrightarrow b :\equiv \neg(a \leftrightarrow b)$.

Exercise 2: Logical Equivalency (1+1+1 points)

Two logical formulae φ, ψ over a set of atoms Σ are logically equivalent ($\varphi \equiv \psi$) iff for all interpretations I of Σ the following holds

$$I \models \varphi \Longleftrightarrow I \models \psi.$$

With the above definition, show or disprove the following equivalencies (e.g. by making truth tables).

(a) $p \land (q \lor r) \overset{?}{\equiv} (p \land q) \lor (p \land r)$

(b) $\neg(p \to q) \overset{?}{\equiv} p \to \neg q$

(c) $\neg(p \land q) \overset{?}{\equiv} \neg p \lor \neg q$

Exercise 3: CNF and DNF (2+2 points)

(a) Convert $\varphi_1 := (\neg p \to q) \to (q \to \neg r)$ into Conjunctive Normal Form (CNF).

(b) Convert $\varphi_2 := \neg(p \to q) \lor ((r \lor s) \to (q \lor t)) \lor (\neg p \to \neg v)$ into Disjunctive Normal Form (DNF).

Remark: Use the known logical equivalencies given in the lecture slides to do the necessary transformations. State which equivalency you are using in each step.

Exercise 4: Logical Entailment (1+2+2 points)

A *knowledge base* KB is a set of formulae over a given set of atoms Σ. An interpretation I of Σ is called a model of KB, if it is a model for *all* formulae in KB. A knowledge base KB *entails* a formula φ (we write $KB \models \varphi$), if *all* models of KB are also models of φ.

Let $KB := \{\neg p \vee q, q \vee \neg r\}$. Show or disprove that KB logically entails the following formulae.

(a) $\varphi_1 := \neg q$

(b) $\varphi_2 := (\neg p \wedge q) \vee \neg(\neg r \vee q)$

(c) $\varphi_3 := \neg(r \wedge \neg q) \vee (\neg q \rightarrow \neg p)$