Theoretical Computer Science - Bridging Course

Tutorial 06

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Exercise 1: Decidable Languages

(a) Show $L_1 := \{\langle A\rangle \mid A \text{ is a CFG that generates } \varepsilon\}$ is decidable. Use that $A_{\text{CFG}} := \{\langle A, w\rangle \mid A \text{ is a CFG that generates } w\}$ is decidable (cf. lecture).

(b) Show $L_2 := \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$ is decidable. Use that $EQ_{\text{DFA}} := \{\langle A, B\rangle \mid A, B \text{ are DFAs and } L(A) = L(B)\}$ is decidable (cf. lecture).

(c) Show that $\{\langle R, S\rangle \mid R, S \text{ are reg. expr. and } L(R) \subseteq L(S)\}$ is decidable. Use that we can derive a DFA A from a regular expression R such that $L(A) = L(R)$ in finite time. Moreover for DFAs A, B we can determine a DFA C with $L(C) = L(A) \cap L(B)$ in finite time.

Exercise 2: Decidability

Consider the following language

$$L := \begin{cases} \{0\}, & \text{if there is life on Mars} \\ \{1\}, & \text{else.} \end{cases}$$

Is L decidable?

Exercise 3: The Halting Problem

(a) Show that the *Halting problem*

$$H := \{\langle M, s\rangle \mid \langle M\rangle \text{ encodes a TM that } \textit{halts} \text{ on input } s\}$$

is semi-decidable.[1]

(b) Show that the Halting problem H is undecidable. Assume that we already know that the *Special Halting problem*

$$H_s := \{\langle M\rangle \mid \langle M\rangle \text{ encodes a TM that } \textit{halts} \text{ on input } \langle M\rangle\}$$

is undecidable.

[1] This definition of the halting problem deviates from the one on the lecture slides, but is also very common.

Exercise 4: Relation between Language Classes

(a) Give a Venn diagram showing the relation between the set of regular, context-free, decidable and semi-decidable languages.

(b) Give an explanation why some of these sets are contained in others.

(c) Show that the subset relations are proper, e.g. by giving a language which is contained in the respective superset but not in the subset.

Exercise 5: Conway's Game of Life

Given a square grid $G = \mathbb{Z} \times \mathbb{Z}$, a cell $(x, y) \in G$ is either alive or dead. Each time step a cell changes its state according to the following rules:

1. A live cell with less than two live neighbors dies.
2. A live cell with two or three live neighbors lives on.
3. A live cell with four or more live neighbors dies.
4. A dead cell with exactly three neighbors becomes alive.

Is Conway's game, which asks for a given start pattern $p_{\text{init}} : G \to \{\text{dead}, \text{alive}\}$ whether a given target pattern p_{target} is ever reached, semi-decidable? Is it decidable?

https://en.wikipedia.org/wiki/Conway%27s_Game_of_Life

https://www.youtube.com/watch?v=vGWGeund3eA