Theoretical Computer Science - Bridging Course

Tutorial 07

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Exercise 1: $\mathcal{O}$-Notation Formal Proofs

The set $\mathcal{O}(f)$ contains all functions that are asymptotically not growing faster than the function f (when additive or multiplicative constants are neglected). That is:

$$g \in \mathcal{O}(f) \iff \exists c \geq 0, \exists M \in \mathbb{N}, \forall n \geq M : g(n) \leq c \cdot f(n)$$

For the following pairs of functions, check whether $f \in \mathcal{O}(g)$ or $g \in \mathcal{O}(f)$ or both.

(a) $f(n) = n,\ g(n) = n^2$

(b) $f(n) = 2^n,\ g(n) = 3^n$

(c) $f(n) = \log_2(n!),\ g(n) = n \log_2 n$ *Hint:* $n! := \prod_{i=1}^{n} i \geq (n/2)^{n/2}$

Exercise 2: Decision and Optimization Problems

Is the following problem an optimization or a decision problem? Transform it into the respective other problem type.

DOMINATINGSET:

- A *dominating set* of graph $G = (V, E)$ is a subset $D \subseteq V$, s.t. for every vertex $v \in V : v \in D$ or v adjacent to a node $u \in D$.
- **Input**: Encoding $\langle G, k \rangle$ of an undirected, unweighted, simple graph $G = (V, E)$ and $k \in \mathbb{N}$.
- **Question**: Is there a dominating set with at most k nodes?

Exercise 2: Decision and Optimization Problems

Is the following problem an optimization or a decision problem? Transform it into the respective other problem type.

VERTEXCOLORING:

- A *vertex coloring* of a graph $G = (V, E)$ is a mapping $c : V \to \{1, \dots, k\}$ such that $c(u) = c(v) \Rightarrow \{u, v\} \notin E$.
- **Input**: Encoding $\langle G \rangle$ of an undirected, unweighted, simple graph $G = (V, E)$.
- **Question**: What is the smallest k for which a valid vertex coloring c of G exists?

Exercise 3: The class $\mathscr{P}$

$\mathscr{P}$ is the set of languages that can be decided by a TM (algorithm) whose run-time can be bounded from above by $p(n)$, where p is a polynomial and n the size of the respective input (problem instance).

Show that that the following languages are in $\mathscr{P}$.

(a) $L := \{w \in \{0,1\}^* \mid w \text{ has even length}\}$
(b) Any regular Language.
(c) 1-DOMINATINGSET $:= \{\langle G \rangle \mid G$ has *dominating set* of size $\leq 1\}$
(d) 2-VERTEXCOLORING $:= \{\langle G \rangle \mid G$ has *vertex cololring* with ≤ 2 col.$\}$