Exercise 1: The Shift Operation  
(4+4 Points)

Consider a Turing machine $M$ that is given an arbitrary input string over alphabet $\Sigma = \{1, 2, \ldots, n\}$ on its input tape. We would like $M$ to insert an empty cell, i.e., $\sqcup$, at the beginning of the tape without removing any symbol on the tape. As an example, the Turing machine is supposed to change the input tape of the form $\langle 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \ldots \rangle$ to $\langle \sqcup, 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \ldots \rangle$. Although this operation is not explicitly defined for a Turing machine, one can consider such an operation as shifting the whole string one cell to the right on the input tape.

(a) Give a formal definition of $M$ to perform the desired operation such that $M$ recognizes the language $\Sigma^*$. 

(b) For $n = 2$, i.e., $\Sigma = \{1, 2\}$, draw the state diagram of your constructed Turing machine.

Exercise 2: Constructing Turing Machines I  
(4+1+2+1 Points)

Let $\Sigma = \{0, 1\}$. For a string $s = s_1s_2\ldots s_n$ with $s_i \in \Sigma$ let $s^R = s_n s_{n-1} \ldots s_1$ be the reversed string. Palindromes are strings $s$ for which $s = s^R$. Then $L = \{ss^R \mid s \in \Sigma^*, a \in \Sigma \cup \{\varepsilon\}\}$ is the language of all palindromes over $\Sigma$.

(a) Give a state diagram of a Turing machine recognizing $L$.

(b) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on its tape.

(c) Describe (informally) the behavior of a 2-tape Turing machine which recognizes $L$ and uses significantly fewer head movements on long inputs than your 1-tape Turing machine.

(d) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes on any of the two tapes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on the first tape.

Exercise 3: Constructing Turing Machines II  
(4 Points)

Let $L = \{\langle w \rangle, \langle w + 1 \rangle \mid w \in \mathbb{N}\}$, e.g., the word $\langle 6 \rangle, \langle 7 \rangle = 110, 111$ is contained in $L$. Design a Turing machine which accepts $L$. You do not need to provide a formal description of the Turing machine but your description has to be detailed enough to explain every possible step of a computation.

Remark: Here $\langle w \rangle$ is the binary encoding of the number $w$, e.g., the number 6 is going to be the string 110.