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Theoretical Computer Science - Bridging Course
Summer Term 2018
Exercise Sheet 6

for getting feedback submit electronically by 06:00 am, Monday, June 11th, 2018

Exercise 1: Constructing Turing Machines *(3+3 Points)*

Construct a Turing Machine for each of the following languages.

(a) $L_1 = \{a^i b^j a^i b^j | i, j > 0\}$

(b) Language L_2 of all strings over alphabet $\{a, b\}$ with the same number of a's and b's.

Remark: It is sufficient to give a detailed description of the Turing Machines. You do not need to give formal definitions.

Exersive 2: Semi-Decidable vs. Recursively Enumerable *(3+3 Points)*

Very often people in computer science use the terms *semi-decidable* and *recursively enumerable* equivalently. The following exercise shows in which way they actually are equivalent. We first recall the definition of both terms.

A language L is *semi-decidable* if there is a Turing machine which accepts every $w \in L$ and does not accept any $w \notin L$ (this means the TM can either reject $w \notin L$ or simply not stop for $w \notin L$).

A language is *recursively enumerable* if there is a Turing machine which eventually outputs every word $w \in L$ and never outputs a word $w \notin L$.

(a) Show that any recursively enumerable language is semi-decidable.

(b) Show that any semi-decidable language is recursively enumerable.

Exercise 3: Halting Problem *(2+2+2+2 Points)*

The *special halting problem* is defined as

$$H_s = \{\langle M \rangle \mid \langle M \rangle \text{ encodes a TM and } M \text{ halts on } \langle M \rangle\}.$$

(a) Show that H_s is undecidable.

Hint: Assume that M is a TM which decides H_s and then construct a TM which halts iff M does not halt. Use this construction to find a contradiction.

(b) Show that the special halting problem is recursively enumerable.

(c) Show that the complement of the special halting problem is not recursively enumerable.

Hint: What can you say about a language L if L and its complement are recursively enumerable? (if you make some observation for this, also prove it)

(d) Let L_1 and L_2 be recursively enumerable languages. Is $L_1 \setminus L_2$ recursively enumerable as well?

(e) Is $L = \{w \in H_s \mid |w| \leq 1742\}$ decidable? Explain your answer!