Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Summer Term 2018
Exercise Sheet 10

for getting feedback submit electronically by 06:00 am, Monday, July 9th, 2018

Exercise 1: Resolution Calculus   *(3+3 Points)*

Considering each of the following cases, first convert the knowledge base (KB_i) and the formula (φ_i) to CNFs. Then, by resolution, show that the knowledge base entails the formula.

(a) $KB_1 := \{(x \wedge y) \to (z \vee w), \ \ y \to x, \ \ (z \wedge y) \to 0, \ \ y\}$
    $\varphi_1 := w \wedge y$

(b) $KB_2 := \{\neg A \to B, \ \ B \to A, \ \ A \to (C \wedge D)\}$
    $\varphi_2 := A \wedge C \wedge D$

Exercise 2: Implication vs. Entailment   *(5 Points)*

Show that $P \models Q \leftrightarrow (True \models P \to Q)$

Exercise 3: Understanding First Order Logic   *(2+2+2 Points)*

Consider the following **first order logical** formulae

$$\varphi_1 := \forall x R(x, x)$$
$$\varphi_2 := \forall x \forall y \, R(x, y) \to (\exists z R(x, z) \wedge R(z, y))$$
$$\varphi_3 := \exists x \exists y \, (\neg R(x, y) \wedge \neg R(y, x))$$

where x, y are variable symbols and R is a binary predicate. Give an interpretation

(a) I_1 which is a **model** of $\varphi_1 \wedge \varphi_2$.

(b) I_2 which is **no model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.

(c) I_3 which is a **model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.

Exercise 4: Truth Value   *(1+1+1 Points)*

Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain (or universe) for the variables consists of:

(a) the positive real numbers,

(b) the integers,

(c) the nonzero real numbers.