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Advanced Algorithms
Problem Set 3

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Exercise 1: Tree with Small Average Stretch

Let $G = (V, E)$ with a distance metric d_G. Moreover, let $w : V^2 \to \mathbb{R}_{\geq 0}$ be a weight function on pairs of nodes. A tree T has average stretch α if

(1.) $\forall u, v \in V : \; d_T(u, v) \geq d_G(u, v)$

(2.) $\sum_{u,v \in V} w(u, v) d_T(u, v) \leq \alpha \cdot \sum_{u,v \in V} w(u, v) d_G(u, v)$.

Show that, given a probabilistic tree embedding $\mathcal{T}$ with stretch $\alpha \in O(\log n)$, you can obtain a tree with average stretch α w.h.p.

Exercise 2: Computing Steiner Forests

Let $G = (V, E)$ with edge weights $w : E \to \mathbb{R}_{\geq 0}$. Furthermore let $\{s_1, t_1\}, \ldots, \{s_k, t_k\} \in \binom{V}{2}$ be a set of pairs of *terminals*. In the Steiner forest problem we are asking for a subset $E' \subseteq E$ with minimal weight $w(E') := \sum_{e \in E'} w(e)$, such that in $G[E']$ each pair s_i, t_i is connected. Use the FRT-algorithm to compute an $O(\log n)$ approximation of a minimal weight Steiner forest E' w.h.p.

Hint: Sample a tree T from a probabilistic tree embedding of G, solve the problem on T, extract a solution for G and compare the result to an optimal solution for G.