exercise 1: tree with small average stretch

let $G = (V, E)$ with a distance metric $d_G$. moreover, let $w : V^2 \rightarrow \mathbb{R}_{\geq 0}$ be a weight function on pairs of nodes. a tree $T$ has average stretch $\alpha$ if

(1.) $\forall u, v \in V : d_T(u, v) \geq d_G(u, v)$

(2.) $\sum_{u, v \in V} w(u, v)d_T(u, v) \leq \alpha \cdot \sum_{u, v \in V} w(u, v)d_G(u, v)$.

show that, given a probabilistic tree embedding $T$ with stretch $\alpha \in O(\log n)$, you can obtain a tree with average stretch $\alpha$ w.h.p.

exercise 2: computing steiner forests

let $G = (V, E)$ with edge weights $w : E \rightarrow \mathbb{R}_{\geq 0}$. furthermore let $\{s_1, t_1\}, \ldots, \{s_k, t_k\} \in \binom{V}{2}$ be a set of pairs of terminals. in the steiner forest problem we are asking for a subset $E' \subseteq E$ with minimal weight $w(E') := \sum_{e \in E'} w(e)$, such that in $G'[E']$ each pair $s_i, t_i$ is connected. use the FRT-algorithm to compute an $O(\log n)$ approximation of a minimal weight steiner forest $E'$ w.h.p.

hint: sample a tree $T$ from a probabilistic tree embedding of $G$, solve the problem on $T$, extract a solution for $G$ and compare the result to an optimal solution for $G$. 