Advanced Algorithms
Problem Set 4
Issued: Friday May 17, 2019

Exercise 1: Multicast Routing

For the Multicast Routing Problem we are given a graph \( G = (V, E, c) \) with edge capacities \( c : E \rightarrow \mathbb{R}_{\geq 0} \) and multi-cast groups \( M_i \subseteq V \) with requirements \( r_i \). We need to output a collection of trees \( P := \bigcup P_i \), where \( P_i \) is a tree which spans \( M_i \) whereas each edge has to reserve capacity \( r_i \) for each tree \( P_i \) that uses this edge. That means, we seek a set of trees \( \bigcup P_i \), such that the maximal congestion:\n\[
\max_{e \in E} \frac{1}{c(e)} \sum_{i : e \in P_i} r_i
\]
is minimized. Show that an \( O(\log n) \) approximation to this problem can be computed efficiently and w.h.p.

Exercise 2: Minimum Bisection Problem

Let \( G = (V, E, c) \) be a graph with an even number of nodes \( |V| \) and edge capacities \( c : E \rightarrow \mathbb{R}_{\geq 0} \). In the Minimum Bisection Problem we are asking for a partition of vertices into two equally sized sets \( (B, W) \) (black and white) with minimal cut (sum of edge capacities between \( B \) and \( W \)). Give an efficient approximation algorithm for the problem, using the tree decomposition designed for multi commodity flow approximation.

*Hint*: You can use that the leaves of trees can be efficiently and optimally bisected.