Exercise 1: Counting Cuts

It is known that a graph with edge connectivity at least $\lambda$ contains at least $\lambda/2$ edge disjoint spanning trees (a result from Tutte and Nash-Williams from the 1960s). Use this result to show that there are at most $O(\lambda n^{2\alpha})$ cuts of size at most $\alpha\lambda$.

Exercise 2: Approximating Cuts in Graphs with Large Expansion

Let $G = (V, E)$ be an unweighted graph for which the following property holds. For all cuts $S$ (w.l.o.g. we assume $|S| \leq |E \setminus S|$ otherwise we switch the roles of $S$ and $E \setminus S$) and some (large) constant $\alpha$ we have that $e(S)/|S| \geq \alpha$. Show that by sampling edges with probability $p := \min(\frac{\epsilon \ln n}{\alpha^2}, 1)$ and assigning appropriate weights, w.h.p. we obtain a subgraph with $\tilde{O}(|E|/\alpha)$ edges that is an $(1 \pm \epsilon)$-approximation of all cuts (for constant $\epsilon > 0$).