Advanced Algorithms

Problem Set 10

Issued: Monday, July 15, 2019

Exercise 1: Evaluating Congestion Approximators

As we have seen in the lecture, we can construct an $m$-congestion approximator based on a maximum spanning tree $T$ as follows ($m := |E|$). For each edge $e \in T$ let $S_e$ be the cut induced by $e$ in the graph. Then we set $R_{e,v} = 1/c_{S_e}$ for all $v \in S_e$ and $R_{e,v} = 0$ for all $v \not\in S_e$, where $c_{S_e}$ is the sum of capacities of edges going over the cut $S_e$. The entries $R_{e,v}$ form a $(n-1) \times n$-matrix $R$. Show that for $x \in \mathbb{R}^n, y \in \mathbb{R}^{n-1}$ we can compute $Rx$ and $R^T y$ in $O(n)$. Assume the capacities of the cuts $c_{S_e}, e \in T$ are known.

Exercise 2: Analysis of the Gradient Descent Procedure

In the lecture we saw that we can reduce the max flow problem to a continuous, unrestricted optimization problem that we solved with the gradient descent method. Show that one step of gradient descent requires $\tilde{O}(m)$ time and one multiplication with $R$ and another one with $R^T$ (where $R$ is the congestion approximator used in the procedure).

Hint: Use the following chain rule for gradients: for $h(x) := g(Ax)$, we have $\nabla h(x) = A^T \cdot \nabla g(Ax)$.