



## Advanced Algorithms

### Problem Set 11

Issued: Tuesday, July 23, 2019

#### Exercise 1: Aggregation in the MPC Model

Assume you are given a number of  $M \in \tilde{O}\left(\frac{N}{S}\right)$  machines (you may freely choose the hidden  $\text{poly}(\log N)$  factor in the number of machines), where  $N$  is the number of *aggregation messages* that are collectively stored by the machines  $M_i$ ,  $i \in [M]$  and each machine  $M_i$  has a memory large enough to store  $S$  messages. We have  $N \gg S$  and  $S \in \tilde{\Omega}(1)$  (for a hidden  $\text{poly}(\log N)$  factor of your choosing). By definition of the MPC model every machine can send and receive at most  $S$  aggregation messages.

Each aggregation message has encoded within it a target machine  $i \leq N$ . Additionally each aggregation message has a value associated with it. The *aggregation problem* is solved as soon as each machine learns an aggregation message that has *minimal* value among all aggregation messages of which it is the target. Carefully formulate a (randomized) algorithm that solves said aggregation problem in  $O(\log N)$  and prove its correctness.

*Hint: You may assume that messages have encoded within them the total number of messages with the same target. Machines have numbers  $1, \dots, M$  that they are aware of. This allows that machine no. 1 computes and distributes public random bits (assume that you have arbitrary public randomness).*