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Algorithms and Data Structures
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Exercise Sheet 10

Exercise 1: Edit Distance

Let $A = a_1 \dots a_n, B = b_1 \dots b_m$ be two words. For $k \leq n, \ell \leq m$ let $A_k = a_1 \dots a_k, B_\ell = b_1 \dots b_\ell$ be the prefixes of A und B. Let $ED_{k,\ell} := ED(A_k, B_\ell)$ be the edit distance of A_k, B_ℓ. Use the dynamic programming algorithm from the lecture to compute $ED_{n,m}$ for the inputs $A = ananas$ und $B = bananen$ by filling a table with values $ED_{k,\ell}$.

Exercise 2: Binomial Coefficient

Consider the following recursive definition of the binomial coefficient

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1},$$

with base cases $\binom{n}{0} = \binom{n}{n} = 1$. Give an algorithm that uses the principle of dynamic programming to compute $\binom{n}{k}$ in $\mathcal{O}(n \cdot k)$ time steps. Argue the running time of your algorithm

Exercise 3: Packaging marbles

We are given n marbles and have access to an (arbitrary) supply of packages. We are also given an array $A[1..n]$, where entry $A[i] \geq i$ is the value of a package containing exactly i marbles. Our profit is the total value of all packages containing at least one marble, minus the cost of packaging, which is i for a package containing i marbles. We want to maximize our profit.

(a) Give an efficient algorithm that uses the principle of dynamic programming to package marbles for a maximum profit.

(b) Argue why your algorithm is correct. Give a tight (asymptotic) upper bound for the running time of your algorithm and prove that it is an upper bound for your solution.