University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn
P. Bamberger, P. Schneider

Algorithms and Data Structures
Summer Term 2019
Sample Solution Exercise Sheet 8

Exercise 1: Check for Cycles

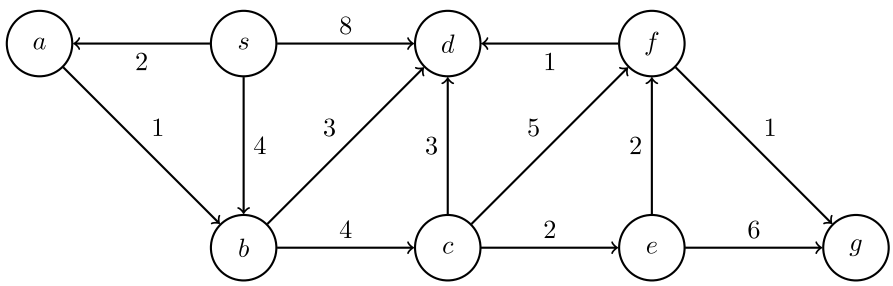
Let $G = (V, E)$ be an undirected graph represented by an adjacency list. Describe an algorithm that tests in $\mathcal{O}(|V|)$ steps whether G has a cycle.

Sample Solution

We do a BFS on the graph. We return "cycle found" whenever a node is processed that has two marked neighbors.
BFS takes $\mathcal{O}(m + n)$. If there is no cycle in the graph, we have $m = \mathcal{O}(n)$ and all nodes are visited after $\mathcal{O}(n)$ steps. If there is a cycle, there must be one in the graph consisting of the first n edges that the BFS looks at. Thus the cycle is detected after $\mathcal{O}(n)$ steps.

Exercise 2: Shortest Paths

Execute **Dijkstras' Algorithm** on the following weighted, directed graph, starting at node s. Write the distances that the algorithm stores in the priority queue after each iteration in the following table.



Initialisation	s	a	b	c	d	e	f	g
$\delta(s,\cdot) =$	0	∞	∞	∞	∞	∞	∞	∞
1. Step $(u = s)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
2. Step $\boldsymbol{(u = \quad)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
3. Step $\boldsymbol{(u = \quad)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
4. Step $\boldsymbol{(u = \quad)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
5. Step $\boldsymbol{(u = \quad)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
6. Step $\boldsymbol{(u = \quad)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
7. Step $\boldsymbol{(u = \quad)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
8. Step $\boldsymbol{(u = \quad)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								

Sample Solution

Initialisation	s	a	b	c	d	e	f	g
$\delta(s,\cdot) =$	0	∞	∞	∞	∞	∞	∞	∞
1. Step $(u = s)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$	0	2	4	∞	8	∞	∞	∞
2. Step $\boldsymbol{(u = a)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$	0	2	3	∞	8	∞	∞	∞
3. Step $\boldsymbol{(u = b)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$	0	2	3	7	6	∞	∞	∞
4. Step $\boldsymbol{(u = d)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$	0	2	3	7	6	∞	∞	∞
5. Step $\boldsymbol{(u = c)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$	0	2	3	7	6	9	12	∞
6. Step $\boldsymbol{(u = e)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$	0	2	3	7	6	9	11	15
7. Step $\boldsymbol{(u = f)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$	0	2	3	7	6	9	11	12
8. Step $\boldsymbol{(u = g)}$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$	0	2	3	7	6	9	11	12