Exercise 1: Drawing DFAs and NFAs \hspace{1cm} \textbf{(8 Points)}

Consider the following three languages over the alphabet \{0, 1\}.

\[ L_1 = \{w \mid |w| \geq 2 \text{ and } w \text{ contains an even number of zeros}\}. \]
\[ L_2 = \{w \mid w \text{ contains exactly two ones}\}. \]
\[ L_3 = \{w \mid w \text{ has an odd number of zeros and ends with 1}\}. \]

First draw a DFA for each of the languages \( L_1, L_2 \) and \( L_3 \). Then, for each of the following languages, provide an NFA that recognizes the given language.

(a) \( L_1^* \)
(b) \( L_3 \circ L_2 \)
(c) \( L_2 \cup L_3 \)

Exercise 2: Regular Languages \hspace{1cm} \textbf{(4 Points)}

Let \( L, L_1, L_2 \) be regular languages. Show that both \( \overline{L} := \Sigma^* \setminus L \) and \( L_1 \cap L_2 \) are regular as well by constructing the corresponding DFAs.

\textbf{Remark: No need for drawing state diagrams. Show how a DFA for the language in question can be constructed presuming the existence of DFAs for \( L, L_1, L_2 \).}
Exercise 3: NFA to DFA  

Consider the following NFA.

(a) Give a formal description of the NFA by giving the alphabet, state set, transition function, start state and the set of accept states.

(b) Construct a DFA which is equivalent to the above NFA by drawing the corresponding state diagram.

(c) Explain what language the automaton recognizes.