Exercise 1: Regular Expressions  

Give a regular expression for each of the following three languages.

(a) \( L_1 = \{ w_1w_2w_3 \mid w_1, w_2, w_3 \in \{a, b, c\}^*, \ w_1 \text{ contains no } a, \ w_2 \text{ contains no } b, \ w_3 \text{ contains no } c \} \)

(b) \( L_2 \subseteq \{a, b\}^* \) is the language of all words that do not have any of the words in \{aaa, aaaa, ...\} as a consecutive substring.

(c) \( L_3 \) is the language, over alphabet \{a, b\}, of all strings not ending with aa.

Exercise 2: The Pumping Lemma: Sufficiency or Necessity?  

Consider the language \( L = \{c^m a^n b^n \mid m, n \geq 0\} \cup \{a, b\}^* \) over the alphabet \( \Sigma = \{a, b, c\} \).

(a) Describe in words (not using the pumping lemma), why \( L \) cannot be a regular language.

(b) Show that the property described in the Pumping Lemma is a necessary condition for regularity but not sufficient for regularity.

   Hint: Use \( L \) as counter example, i.e., show that it can be 'pumped' (in the sense of the pumping lemma), but is still not regular.

Exercise 3: Application of The Pumping Lemma  

Show that the following languages over the alphabet \( \Sigma = \{a, b\} \) are not regular:

(a) \( L = \{a^m \mid m \text{ is a square number}\} \) (\( m \text{ being a square number means that } m = n^2 \text{ for some non negative integer } n \))

   Hint: Use the Pumping Lemma.

(b) \( L = \{a^m b^n \mid m \neq n\} \)

   Hint: Have a look at the languages \( \{a^n b^n \mid n \in \mathbb{N}\} \) and \( a^* b^* \) and use the fact that the class of regular languages is closed under intersection, complement, concatenation and the Kleene star.
Exercise 4: NFAs to Regular Expressions

Consider the following NFA:

Give the regular expression defining the language recognized by this NFA by stepwise converting it into an equivalent GNFA with only two nodes.