Albert-Ludwigs-Universität, Inst. für Informatik
Prof. Dr. Fabian Kuhn
Mohamad Ahmadi

Theoretical Computer Science - Bridging Course
Summer Term 2019
Exercise Sheet 7

for getting feedback submit electronically by 06:00 am, Friday, June 21st, 2019

Exercise 1: Decidability *(7 Points)*

Let Σ be a fixed finite alphabet. Show that the language of deterministic finite automatons (DFAs) on Σ that accept every word is decidable. Formally, show that

$$L = \{\langle A \rangle \mid A \text{ is a deterministic finite automaton with } L(A) = \Sigma^*\}$$

is a decidable language.

Remark: You can use that it is not difficult to construct a Turing machine which tests whether an input is the well formed encoding of a deterministic finite automaton.

Exersive 2: Landau Notation *(2+2+3 Points)*

The set $\mathcal{O}(f)$ contains all functions that are asymptotically not growing faster than the function f (when additive or multiplicative constants are neglected). That is:

$$g \in \mathcal{O}(f) \iff \exists c \geq 0, \exists M \in \mathbb{N}, \forall n \geq M : g(n) \leq c \cdot f(n)$$

For the following pairs of functions, check whether $f \in \mathcal{O}(g)$ or $g \in \mathcal{O}(f)$ or both. Proof your claims (you do not have to prove a negative result $\notin$, though).

(a) $f(n) = 100n,\ g(n) = 0.1 \cdot n^2$

(b) $f(n) = \sqrt[3]{n^2},\ g(n) = \sqrt{n}$

(c) $f(n) = \log_2(2^n \cdot n^3),\ g(n) = 3n$ **Hint:** *You may use that* $\log_2 n \leq n$ *for all* $n \in \mathbb{N}$.

Exercise 3: Sorting Functions by Asymptotic Growth *(6 Points)*

Sort the following functions by asymptotic growth using the $\mathcal{O}$-notation. Write $g <_{\mathcal{O}} f$ if $g \in \mathcal{O}(f)$ *and* $f \notin \mathcal{O}(g)$. Write $g =_{\mathcal{O}} f$ if $f \in \mathcal{O}(g)$ *and* $g \in \mathcal{O}(f)$.

n^2	$\sqrt{n}$	2^n	$\log(n^2)$
3^n	n^{100}	$\log(\sqrt{n})$	$(\log n)^2$
$\log n$	$10^{100}n$	$n!$	$n \log n$
$n \cdot 2^n$	n^n	$\sqrt{\log n}$	n