Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Summer Term 2019
Exercise Sheet 10

for getting feedback submit electronically by 06:00 am, Monday, July 26th, 2019

Exercise 1: Resolution Calculus *(3+3 Points)*

Considering each of the following cases, first convert the knowledge base (KB_i) and the formula (φ_i) to CNFs. Then, by resolution, show that the knowledge base entails the formula.

(a) $KB_1 := \{(x \wedge y) \rightarrow (z \vee w),\ \ y \rightarrow x,\ \ (z \wedge y) \rightarrow 0,\ \ y\}$
$\varphi_1 := w \wedge y$

(b) $KB_2 := \{\neg A \rightarrow B,\ \ B \rightarrow A,\ \ A \rightarrow (C \wedge D)\}$
$\varphi_2 := A \wedge C \wedge D$

Exercise 2: Implication vs. Entailment *(5 Points)*

Show that $P \models Q \leftrightarrow (True \models P \rightarrow Q)$

Exercise 3: Understanding First Order Logic *(2+2+2 Points)*

Consider the following **first order logical** formulae

$$\begin{aligned}
\varphi_1 &:= \forall x R(x,x)\\
\varphi_2 &:= \forall x \forall y\, R(x,y) \rightarrow (\exists z R(x,z) \wedge R(z,y))\\
\varphi_3 &:= \exists x \exists y\, (\neg R(x,y) \wedge \neg R(y,x))
\end{aligned}$$

where x, y are variable symbols and R is a binary predicate. Give an interpretation

(a) I_1 which is a **model** of $\varphi_1 \wedge \varphi_2$.

(b) I_2 which is **no model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.

(c) I_3 which is a **model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.

Exercise 4: Truth Value *(1+1+1 Points)*

Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain (or universe) for the variables consists of:

(a) the positive real numbers,

(b) the integers,

(c) the nonzero real numbers.