Exercise 1: Resolution Calculus  

(3+3 Points) 

Considering each of the following cases, first convert the knowledge base (\(KB_i\)) and the formula (\(\varphi_i\)) to CNFs. Then, by resolution, show that the knowledge base entails the formula. 

(a) \(KB_1 := \{ (x \land y) \rightarrow (z \lor w), \ y \rightarrow x, \ (z \land y) \rightarrow 0, \ y \}\)  
\(\varphi_1 := w \land y\) 

(b) \(KB_2 := \{ \neg A \rightarrow B, \ B \rightarrow A, \ A \rightarrow (C \land D) \}\)  
\(\varphi_2 := A \land C \land D\) 

Exercise 2: Implication vs. Entailment  

(5 Points) 

Show that \(P \models Q \leftrightarrow (\text{True} \models P \rightarrow Q)\) 

Exercise 3: Understanding First Order Logic  

(2+2+2 Points) 

Consider the following first order logical formulae 

\(\varphi_1 := \forall x R(x, x)\)  
\(\varphi_2 := \forall x \forall y R(x, y) \rightarrow (\exists z R(x, z) \land R(z, y))\)  
\(\varphi_3 := \exists x \exists y (\neg R(x, y) \land \neg R(y, x))\) 

where \(x, y\) are variable symbols and \(R\) is a binary predicate. Give an interpretation 

(a) \(I_1\) which is a model of \(\varphi_1 \land \varphi_2\). 

(b) \(I_2\) which is no model of \(\varphi_1 \land \varphi_2 \land \varphi_3\). 

(c) \(I_3\) which is a model of \(\varphi_1 \land \varphi_2 \land \varphi_3\). 

Exercise 4: Truth Value  

(1+1+1 Points) 

Determine the truth value of the statement \(\exists x \forall y (x \leq y^2)\) if the domain (or universe) for the variables consists of: 

(a) the positive real numbers, 

(b) the integers, 

(c) the nonzero real numbers.