

## Distributed Systems, Summer Term 2019

### Exercise Sheet 4

#### 1. Happens Before in Shared Memory

Given are  $n$  processors and  $m$  shared variables. Every processor can access every shared variable with atomic read and write operations (i.e., a process can either read from or write to a shared variable and the system guarantees that such accesses of different processes to the same variable happen atomically). Define a *happens before* relation similar to the one for message passing.

#### 2. Unique Maximal Cut Preceding a Given Cut

Given a schedule  $S$  with a cut  $C$ . Show that there is a unique consistent cut  $C'$  of  $S$  which precedes the cut  $C$ .

#### 3. Happens Before Relation

Let  $S$  be a schedule with events  $a$ ,  $b$ , and  $c$ . Show that if  $a \not\Rightarrow_S b$  and  $a \not\Rightarrow_S c$  holds, then there exists some causal shuffle  $S'$  of  $S$  in which  $b$  and  $c$  occur before  $a$ .

#### 4. Logical Clocks

You are given the clique graph on  $n$  nodes. Find two executions  $A$  and  $B$ , in which each node sends exactly one message to every other node, such that

- the largest Lamport clock value in  $A$  is as small as possible and
- the largest Lamport clock value in  $B$  is as large as possible.

## 5. Clock Synchronization

### Model:

- $n$  processors connected in a path.
- Hardware clock  $H_v$  for each processor, increments are in an interval of  $[1 - \rho, 1 + \rho]$  (clock drift),
- Logical clock  $L_v$  for each processor,
- Message delays are in the interval  $[0, T]$ .

The following (simple) algorithm executed at every processor synchronizes the logical clocks according to hardware clocks and messages from neighbors.

**Algorithm:** If no messages are received the logical clock runs as fast as the hardware clock and the logical clock values are forwarded in every time unit.

If messages from neighbors are received set the logical clock to the maximum of the clock values received from neighbors and the own clock value; forward new clock values to all neighbors immediately.

In all the exercises of this problem set, we consider the synchronous message passing model on a graph, where nodes operate in synchronous rounds and all nodes start a computation together at time 0. We assume that initially, nodes do not know the IDs of their neighbors.

**Exercise:** Give an execution which shows that the maximal skew (the difference between the maximum and minimum (logical) clock value)  $(n - 1)T$  can be obtained.