Exercise 1: Bitstrings without consecutive ones \textit{(10 Points)}

Given a positive integer $n$, we want to compute the number of $n$-digit bitstrings without consecutive ones (e.g., for $n = 3$ this number is $5$, as 000, 001, 010, 100, 101 are the 3-digit bitstrings without consecutive ones).

(a) Give an algorithm which solves this problem in time $O(n)$. Explain the runtime. \textit{(6 Points)}

(b) Implement your solution. You may use the template DP.py. Run your algorithm on the values 10, 20 and 50 and write your results in erfahrungen.txt. \textit{(4 Points)}

Exercise 2: Partitioning \textit{(10 Points)}

Given a set $X = \{x_0, \ldots, x_{n-1}\}$ with $x_i \in \mathbb{N}$, we want to determine whether \textit{there is} a subset $S \subseteq X$ such that $\sum_{x \in S} x = \sum_{x \in X \setminus S} x$ (it is not necessary to compute $S$).

(a) Let $W := \sum_{x \in X} x$. Give a recursive formula $s : \{0, \ldots, n-1\} \times \{0, \ldots, W\} \rightarrow \{\text{True}, \text{False}\}$ such that $s(i, j) = \text{True}$ if and only if there is a $S \subseteq \{x_0, \ldots, x_i\}$ such that $\sum_{x \in S} x = j$. Explain how $s$ can be used to solve the above problem in time $O(W \cdot n)$. \textit{(6 Points)}

(b) Implement your solution. You may use the template DP.py. Run your algorithm on the sets given in set1.txt, set2.txt and set3.txt and write your results to erfahrungen.txt \textit{(4 Points)}