1 2-coloring in paths

Show that there is no randomized distributed algorithm that finds a 2-coloring in paths in $o(n)$ rounds with probability at least 0.9. Assume that $n$ is known and IDs are from $\{1, \ldots, n\}$. Hint: for any possible output, nodes with the same view have the same probability of giving that output.

2 Independent sets in paths

An independent set (IS) is a subset of nodes such that no two neighboring nodes are in the independent set. A maximal independent set (MIS) is an independent set that cannot be extended. Assume that $n$ is known and IDs are from $\{1, \ldots, n\}$. Show that, in paths:

1. it is trivial to find some IS in $O(1)$ time with a deterministic distributed algorithm.
2. there exists an IS with at least $n/2$ nodes.
3. it is not possible to find an IS of size at least $n/2$ in $o(n)$ rounds.
4. there is no deterministic distributed algorithm that finds an MIS in $o(\log^* n)$ rounds.

3 Counting

Assume we are given a path of size $n$ where nodes know an upper bound on the size of the network in $\{n, \ldots, cn\}$ for some constant $c$ (i.e., nodes do not know the exact value of $n$ but only a constant approximation). Show that there is no deterministic distributed algorithm that counts the number of nodes in paths in $o(n)$ rounds. Assume that IDs are from $\{1, \ldots, n\}$.