Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Summer Term 2020
Exercise Sheet 1

In case you wish to get feedback, submit electronically by 12:15, Tuesday, May 19.

Exercise 1: Proof by Induction *(3 Points)*

Prove by induction for any positive integer number $n, n^3 + 2n$ is divisible by 3.

Exercise 2: Which Statement is True? *(2+2+3 Points)*

Let A, B and C be sets in some universal set U. Which of the following statements is always true. Justify.

1. If $A \cap B = A \cap C$, then $B = C$.
2. If $A \cup B = A \cup C$, then $B = C$.
3. $\overline{A \cup B} = \overline{A} \cap \overline{B}$. *Remark:* $\overline{A}$ *is the compliment of* A

Exercise 3: Visiting All Nodes (Part1) *(4 Points)*

A *simple graph* is a graph without self loops, i.e. every edge of the graph is an edge between two distinct nodes. A *complete graph* is a simple undirected graph in which every pair of distinct nodes is connected by a unique edge e.g. a triangle on 3 nodes.

Prove that every complete graph G has a path P that visits all the nodes of G.

Exercise 4: Visiting All Nodes (Part 2) *(6 Points)*

A *directed path* P on n vertices is a simple directed graph whose edge set is the following set of ordered pairs $\{(v_i, v_{i+1}) \mid 1 \leq i \leq n-1 \text{ and } v_i \text{ is a node in } P\}$ i.e. a path in which all the arrows point in the same direction as its steps . We write $P = v_1 v_2 ... v_n$ to denote the directed path P.

Prove that every complete directed graph T has a directed path P that visits all the nodes of T.

Hint: Prove by contradiction. Consider a longest directed path in T *and suppose that this path doesn't visit all nodes in* T*. What happens then?*